THE ELECTRIC AND MAGNETIC CIRCUITS

ALTERNATING CURRENT AND DIRECT CURRENT

By

E. N. PINK, B.Sc., A.M.I.E.E.

Assistant Professor in the Electrical Engineering Department of the Heriot Watt College, Edinburgh

This book will be of considerable help to the student of Electrical Engineering, and others to whom the study of the flow of electricity is of special importance.

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THE ELECTRIC AND MAGNETIC CIRCUITS

ALTERNATING CURRENT AND DIRECT CURRENT

BY

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PREFACE

It is assumed that the student has some knowledge of Electricity and Magnetism, and this book is intended to help the Electrical Engineering Student and others, to whom the study of the flow of electricity is of special importance.

Books on Electricity and Magnetism have such a wide field to cover, that they cannot possibly devote special attention to one part of the subject.

A treatment of the Magnetic Circuit is included, partly because a review of some of the facts of Magnetism is of special importance before commencing the study of alternating currents, and partly on account of its importance in the study of the electrical machines.

It is hoped that the book will be of real help to the student, and that, in making the study of this part of the subject easier and pleasanter, both time and effort will be released for the deeper study of other parts of the subject.

E. N. PINK.

Edinburgh, 1930.

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THE ELECTRIC AND MAGNETIC CIRCUITS

PART I
THE DIRECT CURRENT CIRCUIT

CHAPTER I

INTRODUCTORY

THE study of the electric circuit will no doubt be easier to most if they have in their mind some theory as to the nature of electricity, even if this theory is incomplete and might be considered by some as actually erroneous; still, the theory might be useful in helping the reader to learn certain established facts about the flow of electricity.

Many will have their own theories as to what is going on when electricity is flowing in a conductor. In this book the following theory is made use of, but it must be remembered that this is only employed as a means to an end, and that any other useful theory will serve, considering the present incomplete state of knowledge as to the exact nature of electricity.

Here the atoms of all bodies are supposed to consist of small charges of negative electricity called electrons, surrounding a heavy central portion called the nucleus, and bearing a positive charge.

The arrangement is supposed to be similar to that of the sun and its planets. The nucleus with its positive charge attracts the negative electrons, and the electrons all repel each other. The electrons are apparently held apart from the nucleus by some means. The atom is supposed to be electrostatically neutral, that is, there is as much negative charge in the electrons as positive charge associated with the nucleus. The

electrons are all supposed to be alike in any atom, but the number per atom is different for all the different elements. The different elements have differently made nuclei, and take different numbers of electrons surrounding them to balance their positive charge. The required number of electrons may be only a few, or it may exceed a hundred. For any one element the atoms are supposed all alike.

An insulated body having a negative charge will be looked upon as a body having a surplus of negative electrons on it, and if the body is a conductor these electrons will be considered as free to jump from atom to atom, being most exceedingly mobile and capable of moving through the conductor with immense velocity, even under the action of very small forces. As the surplus electrons would all repel one another, and would distribute themselves accordingly, evidently work would have to be done on them to bring them into close proximity with one another. This helps one to appreciate the meaning of the electrical potential of a body. In bringing up a small charge of the same sign to the charged body, the electrons on the body have all been caused to move closer to one another, and the work done in bringing up the charge has been expended, not only in increasing the charge on the body, but also in increasing the energy associated with each electron, or, in other words, in increasing the potential.

In circuits one is concerned mostly with potential differences, written for short, P.D. There is said to be no P.D. between two insulated conductors, if on joining them with a wire, itself insulated from all other bodies, no redistribution of the charge takes place, that is, no passage of electricity takes place along the wire. If one body has a higher potential than the other, a momentary flow of electricity takes place along the wire, and both bodies then have the same potential. If both had negative charges, some of the electrons of the one with the higher potential would project themselves through the wire on to the body with the lower potential. If one body has a positive charge and the other a negative, then for present purposes it is supposed that the negative electrons project themselves through the wire, impelled by the force of repulsion behind them, and attracted by the positive charge in front of them. If the total negative charge is insufficient to satisfy the deficiency of negative electrons on the positively charged body,

then the positive charge remaining is shared, and both bodies are deficient in electrons.

Where a difference of potential between the ends of a conductor is maintained by a continual supply of electrons, as is often the case, then a steady stream of electrons takes place through the conductor, from the place of higher to the place of lower potential. The electrons evidently give up some of the energy which they had at the place of higher potential in passing to the place of lower potential. Since, by the principle of the conservation of energy, none of this energy is destroyed, it must reappear in some other form there. Work had to be done on the electrons to bring them to the place of higher potential. The meaning of quantity of electricity is more easily understood, since the quantity can be taken to be a definite number of electrons. The electrostatic unit of quantity is too small to be convenient where the flow of electricity is concerned, and a practical unit called the coulomb is adopted, which is 3,000 million times as great. On the other hand, the electrostatic unit of potential is high, and the practical unit, the volt, is used in preference. It takes 300 volts to make up one electrostatic unit. Since, in electrostatic units the potential of a charge is said to be unity, when one erg of work has to be done on unit charge of electricity of the same kind to bring it from infinity up to the charged body, without change of potential, then the work done in conveying one coulomb against a potential of one volt is obviously—

$$\frac{1}{300} \times 3{,}000{,}000{,}000 \text{ ergs}$$

$$= 10 \text{ million ergs}$$

The coulomb corresponds to a very great number of electrons, something of the order of a million million million.

Electromotive-Force and Flow of Electricity

An electromotive-force is thought of as an electric force, produced by some means inside a conductor, which, acting on the atoms of that conductor, is able to pull out some of the negative electrons and drive them up to one end of the conductor, keeping them away from the atoms which, in being robbed, have become positively charged, and attract the detached electrons driven up to the other end of the conductor.

The above definition may seem long, but there is scarcely anything more important in understanding the flow of electricity than the realization of the meaning and properties of an electromotive-force, and a whole chapter might be written about this alone.

No one who has not a proper grasp of the meaning of an E.M.F. and confuses E.M.F.s with differences of potential, will ever have much pleasure in the study of the flow of electricity.

The commonest way in which an E.M.F. may be produced in a conductor is by the rapid motion of that conductor past the poles of a strong magnet or magnets. All dynamos and alternators in power stations produce their E.M.F.s in this way.

No attempt is made here to inquire how the motion of the conductor past the poles produces the electric force in the conductor, as the production of the E.M.F. is an easily proved experimental fact, which can be demonstrated readily in the classroom, with the help of an electromagnet, a piece of wire, and either a galvanometer, milli-voltmeter, or milli-ammeter.

The galvanometer is an instrument which indicates, by the deflection of its moving part, the passage of electricity through it.

The two ends of the wire are joined to the two terminals of the galvanometer, forming a complete circuit, or electrically conducting path.

No electricity flows around this path until the part of the wire between the N. and S. poles is moved, when the galvanometer shows a deflection, indicating the flow of electricity through it.

There is nothing to be seen in the space between the N. and S. poles of the magnet; lines have been drawn there in Fig. 1, but this is only the conventional way of indicating the directions of the magnetic forces everywhere.

The engineer talks of cutting these lines, and says that the cutting of them produces the E.M.F. It is quite true that if the conductor were so moved as not to cut them, there would be no E.M.F., i.e. if in the sketch the conductor were moved horizontally from left to right, instead of across the face of the pole. The more rapid the motion the greater the E.M.F. The E.M.F. only lasts as long as the conductor is cutting the lines, and if the conductor is moved across the face

of the magnet quickly, with the apparatus in Fig. 1, its duration will only be a part of a second.

In a dynamo the poles are much larger and more strongly magnetized. It is arranged that a continuous succession of conductors passes the poles, and thus the E.M.F. can be kept at a steady value as long as the machine runs.

With the present apparatus it could be observed that the direction of the E.M.F. is reversed, if either the direction of

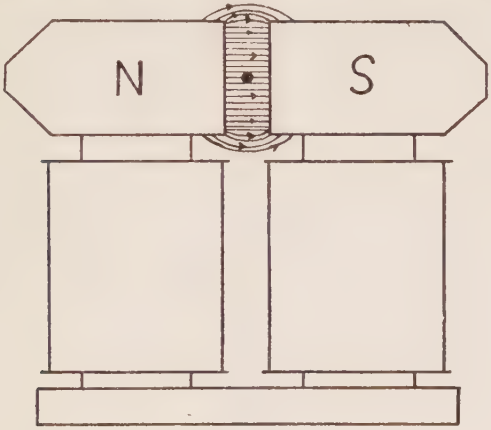

FIG. 1

the motion or the sign of the poles is reversed. One can reverse the poles, making the left-hand one S. instead of N., by reversing the connections of the electromagnet to the battery, or whatever supplies it.

No more force is required to move the conductor across the face of the pole, i.e. make it cut the imaginary lines of magnetic force, than if the magnet were not there, so long as no flow of electricity takes place through the conductor.

If a stream of electricity flows through the conductor, as a result of its motion, there will be found to be a force on the conductor acting against the motion, and work will have to be done to drive the conductor through the magnetic lines against this force. This will be referred to later.

In Fig. 2 a copper conductor is represented, in which an

E.M.F. is being produced by the rapid motion past powerful magnets not shown. Under the action of the electric force, produced somehow in the conductor, some of the negative electrons are detached from their copper atoms and are shot in the direction of the electric force, which we will suppose is from left to right, up to the right-hand end of the conductor.

Supposing, now, that only the middle half of the conductor is cutting the magnetic lines, the freed negative electrons will be found mostly in the right-hand quarter of the conductor. In the left-hand quarter of the conductor will be found a number of atoms of copper, which are short of an electron each, having parted with them under the force of attraction

Fig. 2

from the positively charged group of atoms in the left-hand central portion of the bar.

We can roughly visualize the state of affairs now as depicted in Fig. 2.

An electron situated anywhere inside the bar would be in equilibrium.

For instance, at the centre of the bar, a free electron would be acted on by the electric force, tending to drive it to the right, this force being produced by the cutting of the magnetic lines. Against this force acting on the electron considered, would act a force of repulsion from the negative electrons on the right, and a force of attraction from the positively charged atoms on the left.

In copper the electrons are so exceedingly mobile that, under the action of tiny forces, they shoot along with very large velocities, so a very exact state of balance of the forces tending to drive the electron to the right and the left must exist at all sections of the conductor, which may be considered.

The moment the cutting of lines commences, the conditions considered are instantaneously produced, and all further motion of electrons ceases, until a wire joins the two ends of the conductor—this wire not cutting the magnetic lines.

Some of the electrons at the right-hand end of the conductor

are now shot into the wire by their mutual repulsion, and would instantly pack the wire with the same density as that existing in the right-hand end of the conductor, but since the other end of the wire is joined to the left-hand end of the conductor, the electrons shot into the wire can get away, and they shoot in amongst the positively charged atoms at the left-hand end of the conductor, and are thus absorbed.

The lessening of the density of the packing of the free electrons at the right-hand end of the conductor upsets the equilibrium in the central portion, and the electric force shoots electrons to the right as it did at first, keeping up the density of packing to very nearly what it was before.

Thus we have a mental picture of the flow of electricity under the action of an E.M.F.

An E.M.F., in acting on the electrons which pass, does work on these electrons, giving them energy. After passing through the conductor, in which the E.M.F. is present, they possess energy, by virtue of being present in a densely-packed condition. When they disperse, each is impelled away from its neighbours, and the energy spent in forcing them near to one another reappears in some form or other.

In the case of the conductor whose ends are joined by a wire, which itself does not cut the magnetic lines of force, the energy possessed by each electron, at the crowded end, is spent in giving the electrons a high speed away from the crowded part, and this energy is then turned into heat energy, when the flying electrons collide with electrons belonging to the atoms of the wire.

Thus in this case the mechanical energy spent in driving the conductor past the poles of the magnet reappears as heat energy in the conducting wire and partly in the conductor itself. This is in miniature an electrical power transmission.

Measurement of the Value of an E.M.F.

The value of an E.M.F. produced in a conductor is measured by the value of the difference of potential which is thereby caused. The potential difference, often written P.D., between two charged bodies is the work done in carrying unit quantity of electricity from the one body to the other. The path can be straight from one to the other or curved in any manner, just as the work done against the forces of gravity, in taking

a gallon of water from the top of one hill to the top of another higher hill, depends only on the difference in height.

It makes no difference whether the gallon of water is carried to an intermediate height, exceeding the height of the second hill by, say, 100 ft., for the 1,000 extra ft.-lb. of work done in raising it the extra 100 ft. are all recovered, when the water descends through 100 ft. to the top of the higher hill. The net work depends on the difference in height only.

The potential of the earth is regarded as zero. If an insulated conductor has free negative electrons, its potential is regarded as negative relative to earth and, if it has electrons missing, its potential is called positive.

The difference in potential between two conductors may be measured by connecting each of them, by means of an insulated wire, to different sides of an electrostatic voltmeter.

In the electrostatic voltmeter, there are two sets of small vanes, one fixed and the other capable of rotation about a vertical axis. When there is any difference in potential between the two sets of vanes, which are insulated from one another, they attract one another, and the moving vanes are drawn a little nearer. Some form of control resists this movement, and a pointer indicates by its position the difference of potential—generally directly in volts. The vanes are enclosed in a metal case and the fixed vanes are electrically connected to the case.

If the P.D. between the ends of the conductor cutting the lines of force, when thus measured, should have a value of, say, 1 volt, then the E.M.F. is also said to have a value of 1 volt.

Although an E.M.F. and a P.D. are two very different things, many students confuse them because they both happen to be measured in the same units.

No engineering student would confuse the centrifugal force, acting on the water rotating in the impeller chamber of a centrifugal pump, with the pressure of water at the foot of a stand pipe, due to the height to which the pump is able to force the water.

The centrifugal force is analogous to the E.M.F., and the pressure due to the height of water to the P.D.

CHAPTER II

E.M.F.—POWER—ENERGY

E.M.F.s in Series

The E.M.F. produced in a single conductor of a dynamo is often less than 3 volts. If 500 or 600 volts are desired, 200 or more conductors must be connected one behind the other, or in series as it is called, in such a manner that the 200 or more E.M.F.s add in their effect.

Consider, first of all, two conductors, in which the electric force drives the electrons from left to right in each, and let them be joined, as shown in Fig. 3.

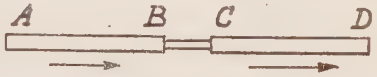

Fig. 3

At B there is an accumulation of electrons due to the E.M.F. between A and B. At C the density of packing of electrons is the same as at B. At D the electrons must be more densely packed than at C, because the difference in density balances the electric force acting through the conductor CD. In other words, the P.D. between A and D is equal to the sum of the P.Ds. between A and B and C and D.

This is very like the multi-stage centrifugal pump, in which the delivery of one stage is connected to the intake of the next stage. Each stage adds to the total pressure, that difference of pressure which it can itself produce.

E.M.F. of Primary and Secondary Batteries

If, for example, a zinc and a copper plate are immersed, without touching each other, in dilute sulphuric acid, a difference of potential of about a volt is produced. This P.D. is called into play by an E.M.F. created in some manner by the different chemical substances in contact in the cell. The zinc dissolves in the acid and provides the energy. The primary cell is of little value, except for very small amounts of energy.

The secondary cell or accumulator is of importance. It has to be charged, by sending a stream of electrons through it from a dynamo. The streams of electrons, in passing through, create, in the case of the lead cell, a coating of lead peroxide, a reddish-brown substance, on the positive plates. When this has been going on for some hours, the cell can be disconnected and it contains an E.M.F. of about 2 volts. It may, on discharge, pass about 80 per cent of the electricity, which was passed through on charge.

Note that chemical energy is stored, not electrical energy.

In charging, the electricity flowed through the cell against the E.M.F., which came into play the moment the charge started. To make the electrons flow against the E.M.F., a P.D., greater than that which the E.M.F. can itself create, must be supplied.

This is a point of importance in learning about the behaviour of E.M.F.s, and is dealt with below.

The lead accumulator gives about 2 volts per cell, whatever the size, small or large. For wireless, accumulators may weigh only a few pounds per cell, whereas in large central stations cells weighing several hundredweights are often used. Both give 2 volts, but the larger the cell, the greater the number of electrons which can be discharged from it per second.

E.M.F.s in Opposition

Fig. 4 shows the connections for a case where one battery of lead accumulators is being charged by a second battery having a greater number of cells, i.e. a greater total E.M.F.

A battery is usually charged from a dynamo, but for present purposes the arrangement shown is more suitable.

On the left of the diagram an ammeter is shown: its purpose is to indicate the quantity of electricity passing per second.

On the right is a representation of a variable resistance. A conductor is said to offer resistance to the flow of electrons when the electrons have difficulty in projecting themselves through it.

At the left-hand end of the bottom battery there is a greater concentration of electrons than the bottom battery could itself create, and in consequence it is obvious that electrons

will be shot through this battery against the electric force, or E.M.F., in the battery.

These electrons will arrive at the right-hand end of the lower battery, where the electrons are not densely packed together; or, in other words, in passing through the lower battery, they have given up a part of their energy, which they had received from the top battery. This energy will be mostly converted into chemical energy in the cells, a little

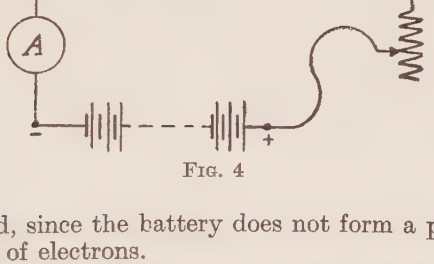

FIG. 4

being wasted, since the battery does not form a perfect path for the flow of electrons.

The battery is thus receiving chemical energy, or is being charged.

The stream of electrons, in passing through the resistance on the right, also gives up some energy: they are more packed together, in front of this resistance, than at the far end of it.

The electrons then shoot into the upper battery and the process of packing them together again is repeated.

Out of this should be seen, as is pointed out below, some very useful facts in connection with E.M.F.s.

Work Done on Electrons by an E.M.F.

As has been seen, there is a difference in the concentration or packing together of the electrons at the two ends of a conductor, in which there is an E.M.F.; and evidently, if a unit quantity of electricity passes through, it either receives or loses a definite amount of energy.

If it is carried through in the direction of the E.M.F., it passes from a place of lesser to a place of greater concentration, and possesses then more energy. This gain of energy is equivalent to the product of the electric force on each electron

passing, the distance over which this force is exerted, and the number of electrons contained in the unit quantity of electricity.

Suppose the E.M.F. to be 300 volts and the quantity carried in a given time to be 3,000 millions of electrostatic units, then the work done on the electricity by the E.M.F. can be easily calculated.

There is a P.D. of 300 volts, i.e. 1 electrostatic unit of potential, between the ends of the conductor in which the E.M.F. is situated. That means, then, from the definition of potential, that 1 erg of work will have to be done on each electrostatic unit quantity of electricity to carry it from the lower potential to the higher potential end by any path whatever. In this case the E.M.F. supplies the force and energy, and the path is through the conductor containing the E.M.F.

$$\text{Work done} = 1 \times 3{,}000 \times 1{,}000{,}000 \text{ ergs}$$
$$= 300 \text{ joules}$$

The erg is such a small unit of energy or work, that the joule, which is 10,000,000 ergs, is more convenient.

The joule amounts to about $\frac{3}{4}$ of a ft.-lb.

As already stated, the electrostatic unit of quantity of electricity is not suitable for such large quantities as are concerned in the flow of electricity, and the practical unit of quantity, the coulomb, which is 3,000 million times as great, is used.

The ampere is the practical unit of the rate of flow of electricity, in coulombs passing per second.

Referring to Fig. 4, if the ammeter indicates 10 amp., that means that electricity is passing through the instrument at the rate of 10 coulombs per second. In the circuit depicted, it is, of course, immaterial where the ammeter is placed in the circuit, since the rate of flow of electricity past all points in the path of flow is the same, for, if not, there would be a continual collection of electrons somewhere in the path. Since the capacity for accommodating electrons is small, this could not take place without instant great rise of potential at the place of accumulation, and there is nothing present able to send electrons to a place of very high concentration.

Note that the ammeter is an instrument for measuring the

quantity of electricity, in coulombs, passing per second, and gives no indication of the velocity of the individual electrons.

It happens that nearly all of the necessary electrical calculations and measurements can be made without a knowledge of the value of this velocity. The quantity of water passing per second over a waterfall corresponds, for water, to the term amperes for electricity.

A flow of electrons is often called a current, so the ampere is the practical unit for measuring a current of electricity.

When a current of 1 amp. flows through a conductor containing an E.M.F. of E volts, in the direction of the E.M.F., energy is given by the E.M.F. to the electricity passing, at the rate of E joules per coulomb, or E joules per second, since the current is 1 amp.

If the current had been I amp., then the E.M.F. would give $E \times I$ joules per second to the electricity passing in that time

It does not matter what kind of an electric circuit is considered, or whether there are other E.M.F.s in the circuit or not: the electricity cannot pass through the conductor containing this E.M.F., in the direction of the E.M.F., without acquiring the amount of energy as above.

For instance, it does not matter how a gallon of water is conveyed from the top of one hill to the top of another: the water will possess an extra amount of energy by virtue of its position at the top of the higher hill, and this will depend only on the difference in heights, being independent of how or why it got there.

Similarly, if the electricity flow through the conductor against the E.M.F., it has to expend some of its energy to push its way through the conductor against the electric force.

Consider, then, the forces on an electron as it passes through the conductor against the E.M.F. The electric force, or E.M.F., tends to drive it backwards and, the concentration of electricity behind it being greater than in front of it, this gives a force forwards. The two forces are sensibly equal, there being an exceedingly small preponderance of one over the other, and the electron moves in this direction.

The concentration forces drive it forward and the E.M.F. resists; the work done by the forces of concentration is done against the electric force or E.M.F., and the energy spent

reappears as mechanical, chemical, or the energy of a magnetic field, depending on what caused the E.M.F.

Naturally the energy equation is as before and, quite generally, it may be stated that a current of I amp. passing through a conductor containing an E.M.F. of E volts gives up or receives there energy at the rate of $E \times I$ joules per second.

Power means rate of doing work, or rate of giving up or receiving energy. So many students mix up the terms power and energy.

The term " power " has, in ordinary language, a different meaning from the technical one. When one talks of a power station, one does not mean a " rate of doing work station "—electrical energy station would be a better name.

Energy has also a technical meaning somewhat differing from its everyday one. Technically, energy means " ability to do work "—work being measured as the product of force times distance exerted in the direction of the line of action of the force.

In this sense no work is done by a man who holds up some heavy article whilst it is being fixed: he may get tired, but he does no work in the technical sense.

It seems such a simple matter to distinguish between power and energy, but many students confuse them.

If, then, the rate of doing work is known, the total amount of work done in a given time can be found by multiplying by that time.

Some 2 h.p. motors may possibly do more work per year than other 10 h.p. motors.

When it is working, the larger one does the work five times as fast, but it may work only a fraction of a day, and the smaller one might work continuously.

The unit of power in electrical calculations is generally the watt or the kilowatt, but one must not think that these are units for electrical power only.

The watt means a rate of doing work of 10,000,000 ergs per second, that is, a force in dynes exerted through a distance per second of such a number of centimetres, that the product is 10,000,000. Evidently 1 joule per second is the same rate of working. The kilowatt is merely a rate 1,000 times greater.

The kilowatt is a somewhat greater rate of working than the horse-power. The horse-power and 746 watts are the same rates of working, expressed in the different units.

Supposing, then, for example, there were a 20 h.p. electric motor supplied with electrical energy from 500-volt mains. Allowing for its efficiency being about 90 per cent, it would require energy at the rate of

$$\frac{20 \times 746}{0{\cdot}9} = 16{,}580 \text{ joules per second}$$

The power required is 16·58 kW.

Every coulomb which passes through the motor, from the higher potential cable to the lower potential cable, gives up V joules of energy, i.e. the same amount of energy, which had to be expended to carry it by means of an E.M.F. E, equal to V, from the low potential to the high potential cable.

In the present example, each coulomb passing will give up 500 joules, so the number which must pass per second is given by

$$\frac{16{,}580}{500} = 33{\cdot}16 \text{ coulombs per second.}$$

The current taken by the motor is 33·16 amp., so long as it gives out 20 h.p.

The energy consumed by the motor per hour would come to an enormous number of joules, viz. $500 \times 33{\cdot}16 \times 60 \times 60$.

Evidently a larger energy unit is wanted, and the watt-hour or kilowatt-hour are used.

The kilowatt-hour is 3,600,000 joules, and is the work done in one hour, at a rate of doing work of 1 kW.

Electrical energy is charged for at so much per kilowatt-hour. The present price in Edinburgh is 3½d. per unit, as the kilowatt-hour is called, for lighting, and five-eighths of a penny for domestic heating.

CHAPTER III

RESISTANCE

Resistance of a Conductor

THE resistance of a conductor, in practical units called ohms, is measured by the value of the ratio of volts to amperes for that conductor, thus,

$$R_{ohms} = \frac{\text{Volts}}{\text{Amps.}}$$

The current may be supplied by a battery. An ammeter measures its value. A voltmeter connected to the ends of the

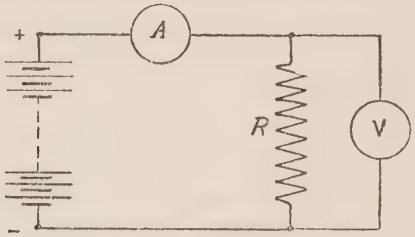

FIG. 5

conductor, whose resistance is to be measured, shows the P.D., and the ratio of voltmeter to ammeter reading gives the value of the resistance in ohms.

In making this measurement there must be no E.M.F. in the conductor, or the ratio of volts to amperes does not give the value of the resistance. The resistance of a battery cannot be measured so simply.

The value of the resistance is independent of the value of the current; care must be taken not to pass so large a current as would overheat the conductor.

If more volts are supplied, the current goes up in proportion, the ratio remaining as before.

Conductors of the same thickness and length do not have the same resistance if made from different metals.

For given dimensions, silver and copper have the least resistance, iron having 6 to 10 times as great a value.

The resistance depends directly on the length, and inversely on the cross-sectional area. A wire of twice the length has twice the resistance. Another having five times the cross-section has one-fifth of the resistance.

Specific Resistance

Evidently, if the resistance of a sample of the material were known, the resistance of any conductor of this material could be calculated. The size of sample taken is of unit length and unit cross-section, and, since the resistance is not the same for different temperatures, the temperature is specified as either 0° C. or 15° C., the latter being useful, as 15° C. is an ordinary temperature.

The resistance of the sample is called the specific resistance of the material.

A complete list of specific resistances of materials will be found in books on Electricity and Magnetism.

For copper it has a value, for centimetre units, of about $1{\cdot}7 \times 10^{-6}$ ohms, or in inch units $0{\cdot}67 \times 10^{-6}$ ohms, at 15° C.

EXAMPLE. Calculate the resistance, at 15° C., of a copper cable 1,000 metres long and 2 sq. cm. in cross-sectional area.

$$\text{Resistance} = \frac{\text{specific resistance} \times \text{length}}{\text{cross-sectional area}}$$

$$= \frac{1{\cdot}7 \times 1{,}000 \times 100}{1{,}000{,}000 \times 2} = 0{\cdot}085 \text{ ohms}$$

EXAMPLE. Find the resistance per mile, at 15° C., of a 0·1 sq. in. copper cable.

$$\text{Resistance} = \frac{0{\cdot}67 \times 1760 \times 36}{1{,}000{,}000 \times 0{\cdot}1} = 0{\cdot}425 \text{ ohms}$$

The specific resistance in inch units is useful where the length is given in yards or miles.

Change of Resistance with Temperature

Conductors composed of pure metals show a higher resistance at higher temperatures. Alloys, such as brass, etc., behave differently for different proportions of their constituents, and one alloy, important for electrical instruments,

shows no change of resistance over a moderate range of temperature. This alloy is called manganin, and is composed of 84 per cent copper, 12 per cent nickel, and 4 per cent manganese.

Returning to the pure metals, any conductor having 1 ohm resistance at 15° C. will have 1·4 ohms at 115° C. Working from the 15° point, its resistance goes up or down 40 per cent. for 100° rise or fall in temperature.

It would have lost 80 per cent of its resistance at 15° C. if reduced in temperature to -185° C.

It is supposed that it would have no resistance if it could be reduced in temperature to the absolute zero of – 273° C.

Iron wire shows a considerably greater change in resistance with change in temperature.

CHAPTER IV

FLOW OF ELECTRICITY

Flow of Electricity—Kirchoff's Laws

GIVEN the values of the resistances and E.M.F.s in a circuit, one can calculate the values of currents everywhere by means of Kirchoff's Laws—

(1) In any closed electric circuit the algebraical sum of the E.M.F.s is equal to the algebraical sum of the products of the resistances and their currents.

(2) When any number of currents meet in a point, the algebraical sum of these currents is zero.

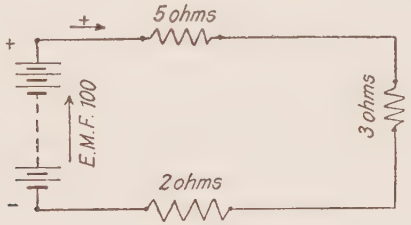

FIG. 6

The second law only applies if the circuit is branched. Consider a simple circuit and apply law (1).

It was conventional to suppose the flow of electricity as taking place from the + terminal of a battery to the - terminal before anything was known of the nature of electricity. Now it may be supposed that the electrons really flow in the opposite direction through the circuit, but the old convention remains and will be adhered to in what follows.

Neglect the resistance of the connecting leads and also that of the battery of accumulators. Lead accumulators have generally a very low resistance.

Since it is the algebraical sum which is to be taken, one must choose a positive direction round the circuit. Either

direction will do; choose it upward through the battery, and the E.M.F. will then have the value, + 100.

Call the current I amp., then

$$+ 100 = 5 \times I + 3 \times I + 2 \times I$$
$$= I(5 + 3 + 2) = 10I$$

The only value of I, which could satisfy this equation, would be + 10.

The current is 10 amp. and flows in the direction chosen as positive.

Ohm's Law.

From the above can be deduced the simpler law, called Ohm's Law, for simple circuits, because out of the above equation it is seen that

$$I = \frac{+ 100}{(5 + 3 + 2)}$$

$$\text{or the current} = \frac{\text{E.M.F.}}{\text{sum of the resistances}}$$

Ohm's Law is a particular case of the general law. Ohm's Law can still be used for a circuit, where some of the resis-

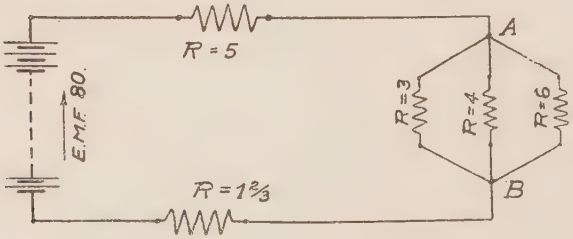

FIG. 7

tances are in parallel instead of in series, so long as there is no E.M.F. in a parallel branch.

The current divides on reaching the three resistances grouped in parallel. Call their resistances r_1, r_2, and r_3.

The P.D. between A and B is unknown. If it had been 1 volt, then, since by definition,

$$\text{Resistance} = \frac{\text{volts}}{\text{amp.}} \quad \therefore \text{ Amp.} = \frac{\text{volts}}{\text{resistance}}$$

and the total current for 1 volt P.D. is

$$\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) \text{amp.}$$

one could imagine the three resistances in parallel to be replaced by one single resistance R, which for the same P.D. would pass the same total current, then

$$R = \frac{\text{volts}}{\text{amp.}} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$\therefore \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

Hence the rule that the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the individual resistances.

There may be any number of these. If the resistances in parallel are all equal, the equivalent resistance is $\frac{1}{n}$th of the resistance of one of the n resistances.

Suppose, in the above example, the three resistances in parallel to have the values 3, 4, and 6 ohms, then

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4 + 3 + 2}{12} = \frac{9}{12}$$

and the equivalent resistance $= \frac{12}{9} = 1\frac{1}{3}$ ohms.

Using Ohm's Law, then the current flowing through the undivided part of the circuit is given by—

$$I = \frac{\text{E.M.F.}}{\text{total resistance}} = \frac{80}{8} = 10 \text{ amp.}$$

One could find how the 10 amp. divide at the parallel resistances, if it were desired, for since resistance is the value of the ratio of volts to amperes for a conductor, the volts across the 5 ohm resistance must be five times the current or 50 volts.

Similarly, $16\frac{2}{3}$ volts are taken up in the $1\frac{2}{3}$ ohm resistance, leaving $13\frac{1}{3}$ volts for the group of parallel resistances, and the currents through each are equal to this voltage divided by the resistances 3, 4, and 6, giving approximately 4·44, 3·33, and 2·23 amp., making together 10 amp.

Circuit with Dynamo

Consider the case of a 2-pole dynamo generating 205 volts and connected to a resistance of 2 ohms, the resistance of each path through the armature winding being 0·1 ohms. No winding on the armature of a dynamo has less than two paths in parallel. The paths have as nearly as possible equal E.M.F.s in them, also equal resistances, and thus should share the total current equally.

The total current cannot be more than about 100 amp., because 100 amp. flowing through the resistance of 2 ohms will take up 200 volts; and the greatest P.D., which the dynamo is capable of supplying, is 205 volts. A few volts will be taken up in sending the current through the resistance of the armature paths.

Supposing the current going out from the machine were 100 amp., Kirchoff's Law could be applied to prove that 50 amp. would come from each armature path.

A closed circuit is formed within the armature, and the law applies to any closed electric circuit, i.e. to any closed electrically-conducting path. Hence it must apply here.

Call the clockwise direction positive, then the algebraical sum of the E.M.F.s is $(+205-205)$, that is, zero, so the IR products must also be zero.

If 50 amp. flowed up each path towards the top brush, these products would both be $50 \times 0{\cdot}1$; and since I in one path is flowing in a clockwise direction, and in the other in a counter-clockwise one, the algebraical sum of the IR products would be zero when both currents are equal.

Another way of looking at the matter would be to consider what current would have to flow in each path to bring the ends of both paths to the common potential, which in each case is equal to the E.M.F. of the path, minus the volts lost in the resistance of that path.

Evidently, if the E.M.F.s differed slightly, one could determine how the two paths would share the current given out

by the machine, and how much current would flow round inside the armature when little or no current were being taken out of the machine. With equal E.M.F.s in each path, no current circulates between the paths.

Now apply Kirchoff's Laws to find the current everywhere in Fig. 8.

From the point of view of Kirchoff's Law there are three paths. One round the armature without going outside the machine; one through the right-hand side of the armature and through the external circuit, as it is called; and the other

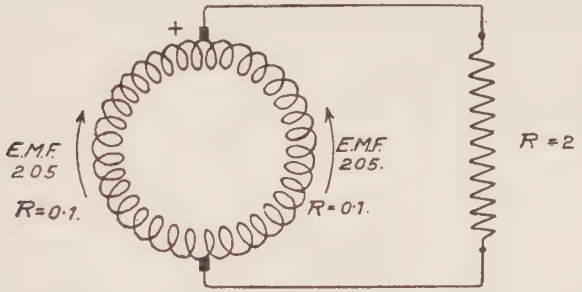

FIG. 8

through the left-hand side of the armature and the external circuit.

Each of these constitutes a closed electrically-conducting path. Parts of the path are common to the various circuits. This does not matter. There is no restriction, but rather the law states "in any closed electric circuit."

Call the current through the 2-ohm resistance I, and consider the circuit through one-half of the armature and the 2-ohm resistance, calling the direction of the E.M.F. positive and the current through each half of the armature $0{\cdot}5I$, then

$$205 = 0{\cdot}1 \times 0{\cdot}5I + 2I$$
$$= 2{\cdot}05I$$

The only value of I which can satisfy the above equation is + 100, and the current flowing down through the 2-ohm resistance has therefore the value 100 amp.

The electrical energy produced per second is $(205 \times 50)2$ joules. Remember that electrical energy is energy possessed

by electricity, in virtue of work being done on the electrons to drive them into close proximity to one another, against the forces of repulsion, which they all exert. The resistance of each armature path, for the sake of clearness, might be thought of as concentrated at the end of the path, just before the brush. Each coulomb, which passes through the E.M.F., receives 205 joules of energy supplied by the E.M.F., which in turn is supplied with mechanical energy of equivalent amount. The electricity, flowing through the 0·1 ohm resistance of one armature path, now loses 5 joules per coulomb, since the potential drops by 5 volts, when electricity is passing at the rate of 50 coulombs per second.

Some of the electrical energy is given up there, and changed into heat energy in this resistance.

Each coulomb passing away from the machine now possesses 200 joules of electrical energy.

The electricity gets away as fast as it can through the 2-ohm resistance, escaping, as has been shown, at the rate of 100 coulombs per second, and arrives into the bottom connection with the electrons far apart, having given up the energy which it had received from the dynamo.

Each coulomb which has passed through the dynamo was given 205 joules of electrical energy; and, in passing on, 5 joules were given up as heat in the armature path and 200 in the 2-ohm resistance, also turned into heat energy.

Rate at which Energy is Given Up in a Resistance

The rate at which energy is given up in a resistance of value R ohms, traversed by a steady current of I amp., is evidently I^2R joules per second, because the potential difference is $I \times R$ volts and the number of coulombs per second is I. Further, the work done in carrying a coulomb from one place to another against the electric forces is all given up again, when the electricity is released and makes its way, in any manner whatever, back again. The same is true of a gallon of water conveyed to the top of a hill. The work done on it will be all given out again in some way or another, when the water comes down, and when the water is at the bottom of the hill again, it has lost all the energy which it had by virtue of being situated at the top.

CHAPTER V

CIRCUITS

Battery being Charged by a Dynamo

CONSIDER the circuit shown in Fig. 9, in which a battery is being charged by a dynamo, the values of the E.M.F.s and resistances being given on the diagram.

In charging a battery of accumulators, the E.M.F. of the dynamo must be opposed to that of the battery and have a

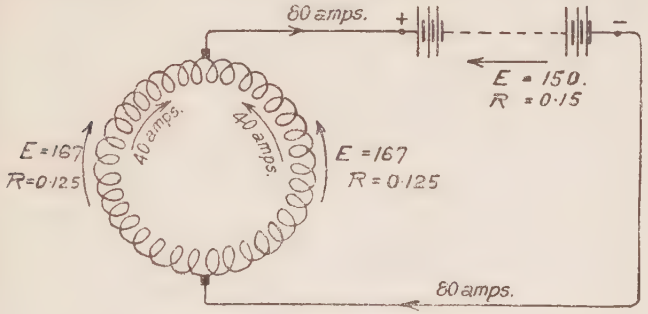

FIG. 9

somewhat greater value. The E.M.F. of a lead accumulator rises during the charging, and finally reaches 2·5 to 2·6 volts towards the end of the charging. This battery contains 60 or more cells in series, according to the supposed state of charge.

It can be assumed that the current in each of the two armature paths is half the total current given out by the machine, this point having been dealt with previously. Taking a circuit through the battery, and returning through either of the armature paths, also calling the total current I amp., and the positive direction round the circuit, that from left to right through the battery, one gets the equation

$$(167 - 150) = 0{\cdot}15I + 0{\cdot}125 \times 0{\cdot}5I$$
$$17 = 0{\cdot}2125I$$

From this I evidently has the value + 80, meaning that 80 amp. pass through the battery against the E.M.F. This E.M.F. has chemical origin, and experiment proves that this energy, viz. 150×80 joules each second, will reappear as chemical energy. The energy per second expended in the resistance of the battery, viz. $80 \times 80 \times 0{\cdot}15 = 960$ joules, is wasted in heat.

Another View of Kirchoff's Law

Of course, Kirchoff's Law, as illustrated by the above examples, really only means that, starting from some given point, such as the – brush in the last circuit (Fig. 9), and following one's way round some closed path, back to the starting point, the sum of the rises in potential on the way must be exactly equal to the falls.

One might compare it with a walk from a certain point amongst the hills, finally returning to the same point, then obviously the sum of the heights, in feet, climbed on the way, must be equal to the sum of the descents in feet.

Starting from the – brush, through the left-hand armature path, brings one to a point 167 volts in potential above the starting point, there is then a 5-volt fall through the resistance of the armature path, bringing one through the + brush to the + terminal of the battery. Here, in passing against the E.M.F. of 150 volts, a point 150 volts lower in potential is reached, since the effect of the 150 volts E.M.F. is to make the one end of the conductor, in which it is, 150 volts higher in potential than the other in the direction of the E.M.F.

A further fall in potential occurs because of the 80 amp. flowing through the 0·15 ohm resistance of the battery: 12 volts are accounted for in this way.

If the potential of the – brush had been called zero, and it could have been made zero by a connection to earth, the changes of potential in the path discussed would have been 0 to + 167, + 167 to + 162, + 162 to + 12, and + 12 back to 0, the starting point.

This point of view is useful in helping one to appreciate the law and its application. It makes it obvious that any closed conducting path may be taken; that in the more complicated circuits which follow, it is of no importance which battery sends which current through any resistance: the only

thing which matters is the value and direction of the current and the value of the resistance through which this current flows.

A More Complicated Circuit

Consider the more complicated circuit shown in Fig. 10.

Neglect, for the sake of simplicity, the small resistance of the lead accumulators and the connecting wires.

The values and directions of the currents have been inserted

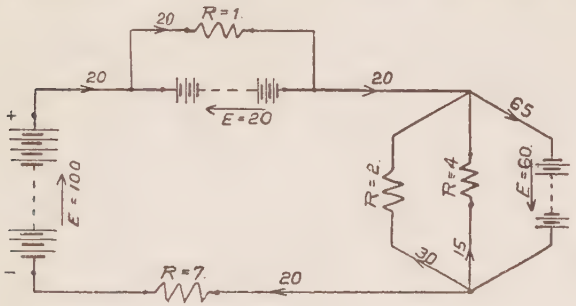

Fig. 10

in the diagram. The explanation of how the values were determined is as follows—

Any path, and there are many possible paths in the diagram, will give a useful equation, so long as it leads through only one of the resistances and one or more E.M.F.s. If a path leading through two resistances is chosen, the currents being different in each, an equation with two unknowns is obtained. With the help of another or simultaneous equation, one might be able to find the solution, but should not resort to that whilst so many simple equations offer themselves. Therefore choose any path leading through any one resistance, and one or any number of E.M.F.s.

For instance, take the 1-ohm resistance at the top, and consider the path through it and the 20-volt battery, then

$$20 = I \times 1 \qquad \therefore I = +20$$

The + sign indicates that the 20 amp. are in the same sense round this circuit, as the 20 volts E.M.F. It should not be

assumed that the 20 amp. come out of the 20-volt battery. There is nothing, so far, to indicate where they do come from. The equation merely shows that 20 amp., from somewhere, must flow through the resistance. The value 20 amp. could have been found otherwise. One could have said that the 1-ohm resistance is directly connected across the terminals of a 20-volt battery, and hence the P.D. between the two ends of the resistance is 20 volts; and since by definition of resistance, ratio of volts to amperes, the volts being 20, the amperes must also be 20, for a ratio unity.

One should have a mental picture of a stream of electrons passing through a resistance. The electrons are more densely packed together in front of the resistance than at the other end of it. Consider an electron anywhere on its way through the resistance. The electrons behind it are a little more closely packed together, than in front of it, in the direction of motion. Consequently there would be a small resultant force driving it forward. It increases its velocity, but probably soon collides with an electron belonging to an atom and loses some of its energy, then starting all over again, finally reaching in, to us, an inconceivably short time, the lower potential end of the conductor considered, having given up some of the energy which it had on entering the conductor, because it is now at a place of less density of packing, i.e. of less potential.

The value of the current, through the 7-ohm resistance at the foot of the diagram, can easily be found, for a path having no other resistances in it is available, viz. that through the 100-volt battery, then the 20-volt, and finally the 60-volt battery. Taking the + direction upwards through the 100-volt battery, the equation is

$$100 - 20 + 60 = 7 \times I$$

$$\therefore\ I = +20$$

The same quantity of electricity per second must come out of the battery as went in. No electrons can continuously come out of any battery unless an equal quantity come back to it.

The current coming out of the battery at the top is therefore fixed at 20 amp.

Kirchoff's second law fixes the current through the 20-volt battery at zero.

The algebraical sum of the currents meeting at any point in a circuit being zero, call the arriving currents positive and the leaving ones negative, then

$$+ 20 - 20 + \text{unknown} = 0$$
$$\therefore \text{ unknown current } = 0$$

In a similar way the current, 65 amp., is found for the 60-volt battery, after having determined the currents through the 2-ohm and 4-ohm resistances, as 30 and 15 amp., then

$$+ 20 + 30 + 15 + \text{unknown} = 0$$
$$\therefore \text{ the unknown current} = -65 \text{ amp.}$$

The minus sign merely indicates that the current flows away from the junction point.

The current, through the 2-ohm resistance, was got by considering the path through it and the 60-volt battery, this being the simplest path. Similarly, the path through the 4-ohm resistance and the 60-volt battery is considered, and the only possible value of the current through the resistance part of this path is seen to be 15 amp.

As an example of a slightly more complicated equation, leading to the same result, one could, for the 4-ohm resistance, have taken the path through the 7-ohm resistance, the 100-volt battery, the 1-ohm resistance, and finally through the 4-ohm resistance, in which the current is to be determined, the currents in the other resistances being known.

The equation would have been—

$$100 = (+ 20 \times 7) + (+ 20 \times 1) + I \times 4$$
$$= 140 + 20 + 4I$$
$$\therefore -60 = 4I \text{ and } I \text{ must be } -15$$

This result is as before, but not so simply obtained.

A further useful illustration is afforded by considering the path up through the 2-ohm resistance and down through the 4-ohm resistance.

Testing with the values of currents found, since there is no E.M.F. in the path, the equation is

$$0 = (+ 30 \times 2) + (-15 \times 4)$$
$$= (+ 60 - 60) = 0$$

Value of Study of Complicated Circuits

Some students ask, when being shown such diagrams as the foregoing, whether similar circuits are ever met with in practice.

The answer is " probably not," but their extreme usefulness lies in their suitability for teaching the student to understand the electric circuit easily and completely. Before the author adopted this method, he found it difficult or impossible to give the student a real grip of the conditions of the electric

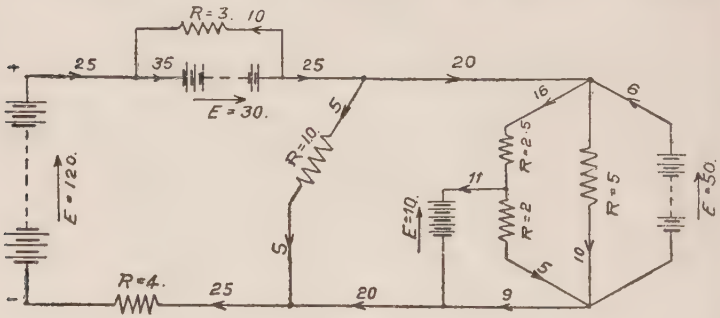

Fig. 11

circuit. Now this is easy and amusing; and, when it comes to the study of the alternating-current circuit and three-phase circuits, the student finds that three-quarters of the difficulties have vanished. This is surely an object well worthy of attainment.

Another Circuit

The circuit in Fig. 11 is a little more complicated, but the solution is not difficult: it follows on the lines of the previous example.

In working out the above, first find the value of the current in every resistance. Taking the path through the 120-volt, the 30-volt, and the 50-volt battery, and returning along the bottom through the 4-ohm resistance, one finds that the algebraical sum of the E.M.F.s is 100 volts, and the current is therefore 25 amp.; also since this current goes into the bottom of the 120-volt battery, 25 amp. pass out at the top. The

current for the 3-ohm resistance must obviously be 10 amp. from right to left; the current through the 30-volt battery follows from Kirchoff's second law.

The 10-ohm resistance and the 50-volt battery give a path, for which a simple equation is obvious, and the current is 5 amp. down. Since 35 amp. come through the 30-volt battery, and 10 of them go round through the 3-ohm resistance, 25 amp. must go on; and since it is obvious that 5 amp. go down through the 10-ohm resistance, 20 amp. must go on from left to right along the top.

The value 16 amp., through the 2·5-ohm resistance, is found by considering the circuit through the 50-volt and 10-volt batteries, giving $50 - 10 = 40$ volts.

The 2-ohm resistance is across the terminals of a 10-volt battery, and the current must be 5 amp. Similarly the current through the 5-ohm resistance is 10 amp.

A current of 20 amp. arrives along the top, and meets currents of 16 and 10 amp. going away, so the missing 6 amp., by the second law, come up through the 50-volt battery.

To determine the current through the 10-volt battery, one can say that 16 amp. come down and 5 go on through the 2-ohm resistance, so 11 must go through the 10-volt battery.

The 5 amp. and 10 amp. coming down, combined with the 6 amp. going up through the 50-volt battery, lead to the value 9 amp. going away towards the left, making 20 amp., after meeting the 11 amp. coming down through the 10-volt battery.

The above is a good example of the use of Kirchoff's laws.

CHAPTER VI

MULTIPLE CIRCUITS

Circuits of Three-phase Style

THE following circuit is different, and is useful in leading up to the study of the three-phase alternating-current circuits.

In the three-phase circuits, the E.M.F.s would be produced by a machine, and these E.M.F.s go through a continuous

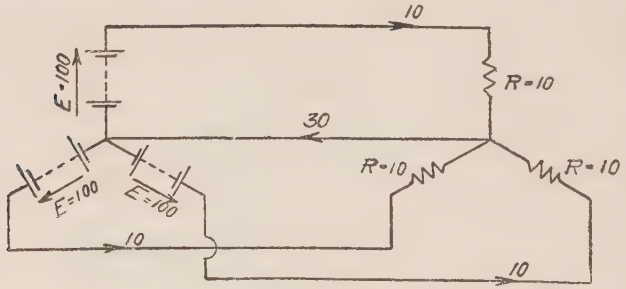

FIG. 12

cycle of changes in value and also direction. The junction point is called the star point. The three resistances represent what is called the load, because the machine does work on it. There is also a star point on the load, and each branch from it can be called one of the arms of the load.

In the present case, the load is a balanced one, i.e. all three arms are alike.

There are quite a number of possible electric paths to which Kirchoff's Law may be applied, but the easier ones are those three, each of which consists of a battery, a corresponding arm of the load, and the return wire between the star points.

The currents out from each battery, along the line, will be seen to be 10 amp., and these meeting at the star point of the load give 30 amp. returning along the middle wire.

In three-phase work, the three E.M.F.s are at no instant all equal. At any instant their algebraical sum is equal to zero, reckoning them positive when acting outwards from the star point.

Algebraic Sum of E.M.F.s Zero

In the two following examples, the algebraical sum of the E.M.F.s is zero.

Apply Kirchoff's Law in turn to each of the three circuits containing an E.M.F., the corresponding load arm, and the

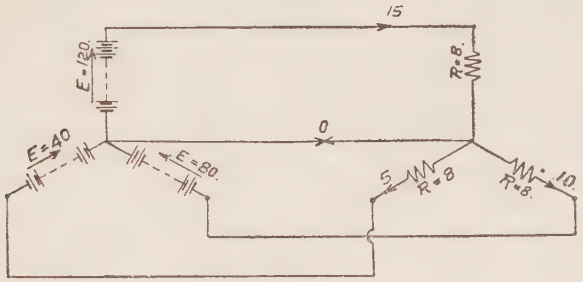

FIG. 13

middle wire; thus the values 15, 5, and 10 amp. are found. No current flows along the middle wire, and it could be removed without altering the conditions.

In connection with these diagrams, it should be noted that any two of the arms cannot be treated as made up of resistances in series.

The real meaning of "in series," for electrical work, is that whatever electricity flows through one piece of apparatus is bound to flow on through the second piece of apparatus connected in series with it.

In Fig. 13, electricity passing through the top 8-ohm resistance, and reaching the star point, is not all bound to pass through either of the other 8-ohm resistances, as it has an alternative path, therefore there is no question of one being considered as connected in series with another.

In these circuits, if the algebraical sum of the threeE.M.F.s is zero and the three-load arms are all alike, then it will always be found that no current returns by the middle wire.

Unbalanced Load

The solution of the above is more difficult. Call the currents in the resistances 12, 8, and 4, I_{12}, I_8, and I_4, reckoning them positive if they flow towards the star point. If then, any of them come out, after evaluation, with a minus sign, it will be known that it flows in reality away from the star point.

By Kirchoff's laws,

$$40 + 120 = 12I_{12} - 4I_4 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$80 + 120 = 12I_{12} - 8I_8 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$40 - 80 = 8I_8 - 4I_4 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$I_{12} + I_8 + I_4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From (4), $I_{12} = -I_8 - I_4$,

substituting this value of I_{12} in (1) and (2), one gets

$$160 = -12I_8 - 12I_4 - 4I_4 = -12I_8 - 16I_4$$

$$200 = -12I_8 - 12I_4 - 8I_8 = -20I_8 - 12I_4$$

Multiplying one equation throughout by 3 and the other by 4, one sees that

$$480 = -36I_8 - 48I_4$$

$$800 = -80I_8 - 48I_4$$

Subtracting,

$$-320 = 44I_8 \qquad \therefore \; I_8 = -7{\cdot}273 \text{ amp.}$$

Inserting this value in (3),

$$-40 = 8(-7{\cdot}273) - 4I_4$$

$$\therefore \; 10 = 2 \times 7{\cdot}273 + I_4$$

$$\therefore \; I_4 = -14{\cdot}546 + 10 = -4{\cdot}546 \text{ amp.}$$

These values have been inserted in the diagram.

The value of the third current is now obviously 11·819 amp.

Test of Results

The truth of the above solution can be tested by considering the changes in potential round any closed path in Fig. 14.

Three electrostatic voltmeters show the P.D. for each resistance. For convenience, the potential of the star point

at the left-hand end is considered zero. It may be connected to earth if desired.

Starting in each case from this point, trace the changes of potential. For instance, going through the 120 battery on to the top line, the potential there must be 120 volts, and in descending through the 12-ohm resistance to the star point, the potential falls off by 141·828 volts; and if the path is continued through the 4-ohm resistance, a further fall of 18·284 volts occur, showing that the potential of this line is (160 – 120), i.e. 40 volts lower than that of the starting point:

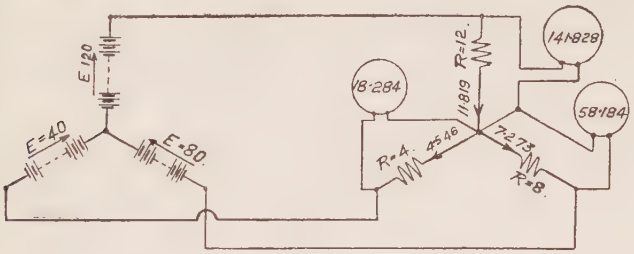

Fig. 14

as the path is completed through the 40-volt E.M.F., in its direction of action the potential will be raised 40 volts, bringing it back to the value zero. This shows the solution, so far, to be correct, small fractions of a volt being neglected in comparison with numbers exceeding 100 volts. If the currents had been found to a greater number of decimal places, there would have been a more exact agreement.

Examination questions have sometimes been set asking for the P.D. between the star points. This could have been easily found from the figures already obtained. Passing again from the one star point to the other through the 120-volt E.M.F. and 12-ohm resistance, the change of potential is + 120 – 141·828, so the P.D. between the star points is 21·828 volts.

If the path through the 40-volt E.M.F. and the 4-ohm resistance had been selected, the changes would have been – 40 and + 18·284, giving the P.D. as 21·716. The small discrepancy is due to the slight inaccuracy of the current values.

When tracing these changes of potential, for instance, as immediately above, one can avoid possible mistakes of signs

by thinking of the electrons as more closely packed at that end of a battery in the direction of the E.M.F., and also as more closely packed in front of a resistance in the direction of the current. With the E.M.F., no account of the direction of the current need be taken.

Alternative Method of Solution

Consider the circuit in Fig. 15.

The currents through the resistances must, by Kirchoff's Law, be as shown in Fig. 15, necessitating a current of 11 amp. through the battery.

Returning now to Fig. 14, there is an alternative method

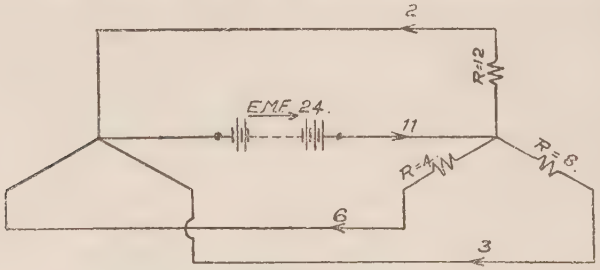

FIG. 15

of finding the solution, which has some importance for alternating currents.

Alternating current is often written A.C. and direct current D.C. One should be familiar with the meanings of A.C. and D.C.

The method is as follows—

(*a*) Imagine the star points temporarily joined, and find the current and its direction along this middle wire.

(*b*) Next imagine the three E.M.F.s on the left reduced each to zero, and a battery of any convenient voltage introduced into the middle wire. Find under these conditions the current in the middle wire, as has been done for Fig. 15.

(*c*) By simple proportion, find the value of the E.M.F. which will cause the middle-wire current in case (*b*) to be exactly equal and opposite to that in (*a*).

(*d*) When the above E.M.F. acts in the middle wire in conjunction with the three original E.M.F.s in the star, no current

will flow along the middle wire, but the finding of the values of the currents in the resistances is now very simple.

Applying this to the circuit of Fig. 14,

(*a*) 10 amp. along middle wire from left to right.

(*b*) The E.M.F. of 24 volts in Fig. 15 will do for this, giving 11 amp. from left to right.

(*c*) Evidently 24 volts E.M.F. is nearly correct, but slightly too large. By proportion, $\frac{11}{11} \times 24$, or 21·82 volts, will give the 10 amp. Acts from right to left.

(*d*) Considering one of the circuits through a main E.M.F. and back through the middle wire, one gets $120 + 21{\cdot}82$

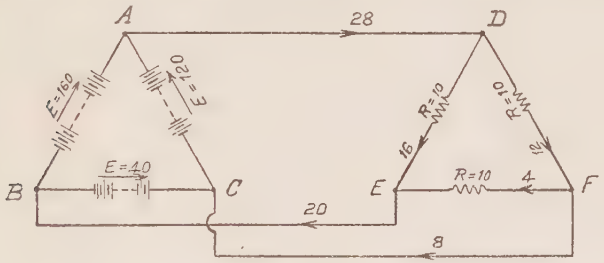

Fig. 16

$= 12I$, whence the value of the current through the 12-ohm resistance is 11·82 amp., almost precisely as previously found. The other values will be found to agree also, and as no current will flow along the middle wire, it might be removed.

The above method is chiefly useful for three-phase A.C. unbalanced loads, and will be referred to later in that connection.

Mesh Connections

Consider the circuit of Fig. 16. It also has considerable importance in A.C. work.

In three-phase work this is referred to a "mesh" connection, the connection already described being called "star."

In A.C. work the instantaneous values of the three E.M.F.s balance against one another in the mesh, and no current would flow in the mesh, unless the machine were connected to a load.

The load chosen consists of three 10-ohm resistances connected in mesh.

One can apply Kirchoff's laws, or the solution can be seen from the following arguments.

Since the points D and E of the load are directly connected to the points A and B of the supply by connections thick enough to prevent any appreciable drop of volts, therefore ED must take the same current, as if it were directly connected to the 160-volt battery alone. The current from D to E is therefore 16 amp. Similarly, the currents from F to E, and from D to F, are 4 and 12 amp.

In each case the resistance concerned gets a certain P.D. between its ends, and nothing but a miracle could prevent electricity from projecting itself through these resistances at the given rates.

The line currents are found from the second of Kirchoff's laws.

The resistance of the batteries has so far been neglected, and thus the currents through the batteries in this case cannot be completely determined. It can be seen that the 160-volt and the 120-volt batteries will give out 28 amp. between them, but without taking their resistances into account, one cannot determine how they share the 28 amp.

If the small resistance of the batteries had been taken into account, it would have been found that the batteries each carried a current of the same value as that in the corresponding load arm, i.e. the 160-volt battery would have supplied 16 of the 28 amp., the remaining 12 being supplied by the 120-volt battery.

CHAPTER VII

MOTOR EFFICIENCY

Resistance of Connections Considered

In the circuit in Fig. 17, the resistances of all parts are taken into account.

Find the currents everywhere, and the volts lost in the cables. Find also the gross mechanical power which the armature of the motor is receiving.

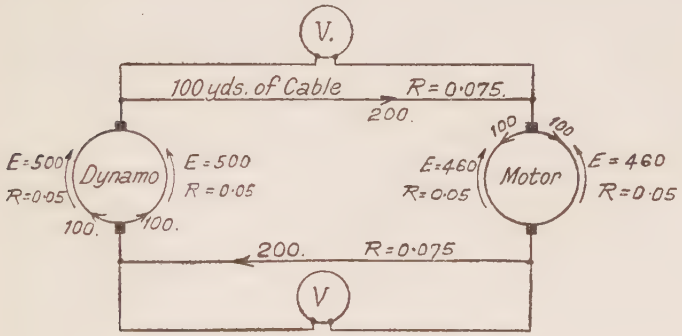

Fig. 17

Applying Kirchoff's Law to a circuit through the left-hand armature path of both dynamo and motor, and calling the cable current I, then

$$500 - 460 = 0{\cdot}05\frac{I}{2} + 0{\cdot}075I + 0{\cdot}05\frac{I}{2} + 0{\cdot}075I$$

$$\therefore\ 40 = 0{\cdot}2I \qquad \therefore\ I = 200 \text{ amp.}$$

The E.M.F. generated by the motor is 460 volts, and each coulomb forcing its way through here, against this E.M.F., will give up 460 joules of its energy, and this energy reappears as mechanical energy communicated to the motor.

The gross mechanical power is $460 \times 100 \times 2$ or 92,000

watts. It is called gross because not quite all of this 92 kW. is available at the pulley of the motor, since friction, windage, and iron losses all cause a drag on the armature; and what-

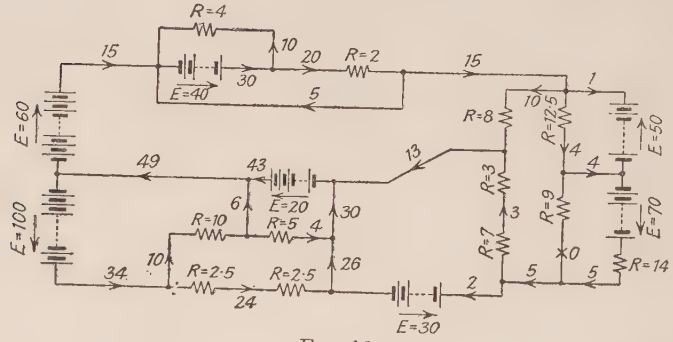

FIG. 18

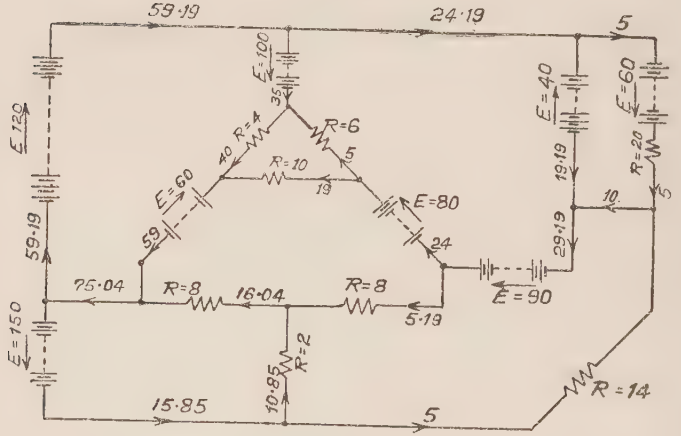

FIG. 19

ever drives the armature round has to supply the loss of useful energy.

The volts drop in each 100 yd. of cable, in Fig. 17, is $I \times R$, that is, $200 \times 0{\cdot}075 = 15$ volts.

The voltmeters, indicated in the sketch, would each show

15 volts P.D. This total loss of volts, with 200 amp. flowing in the cables, is 30 volts.

Current Taken by a Motor for Given H.P.

It is easy to calculate the current taken by a motor for a given h.p., if the efficiency of the motor, for the loading considered, is known.

EXAMPLE. Find the current taken on full load by a 500-volt, 100-h.p., D.C. shunt motor, its efficiency being 0·92.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{100 \times 746}{500 \times I} = 0{\cdot}92$$

Solving this simple equation, it will be found that the full-load current, I, has the value 162·5 amp.

The above little equation has to be used very often, and it has been noticed that the greater number of mistakes made by students are traceable to their no longer bothering to write down the little equation in words; this is evidently necessary.

Two Complicated Circuits

The other two circuits are given as additional examples for the student. He is recommended to make a neat copy to a larger scale, without, however, inserting the values or directions of the currents. He can then fill in these values by himself, and later compare with those given in the book to assure himself that he has been able to do the work correctly. Commence by finding the currents through all the resistances, and the other values of currents will follow from the use of Kirchoff's second law.

PART II
THE MAGNETIC CIRCUIT

CHAPTER VIII

MAGNETS AND FIELDS

Introduction

SOME knowledge of magnetism and the magnetic circuit is essential to the electrical man. Reading up the matter in books on Electricity and Magnetism is good, but since every fact has to be dealt with there, quite irrespective of its importance to the electrical student or engineer, it is far more difficult and much more laborious to sift out and collect together the important facts. This has been done here. No one who has an insufficient knowledge of the following facts will have much pleasure in studying the electrical machines or their design.

Bar Magnets

A bar magnet is made by magnetizing a hard steel bar. It is often called a permanent magnet as contrasted with an electromagnet, which loses its magnetism when the current used to excite it is switched off. If either end of the bar magnet be dipped into a box of small iron nails, a number of these will adhere to either end, but they will not adhere to the middle of the bar.

The forces seem to come from points or regions just inside either end of the bar: the regions or spots from which the forces seem to emanate are called the poles of the magnet.

Every bar magnet has two poles: both attract the small iron nails equally well, but the two poles are different, since, when the magnet is suspended horizontally from a thread, it always comes to rest with the same end pointing towards the north. The end tending to point towards the north is called the north pole, and the other end the south pole.

When two bar magnets are brought near to one another, it is found that there is a force of attraction between unlike poles and a force of repulsion between like poles: thus two norths repel, also two souths, but a north and a south pole attract one another.

Magnetic Force

A magnetic force is the force which one magnet exerts on another magnet. A magnet is not capable of exerting an attractive force on a piece of wood or glass. The force can be exerted across an intervening space without there being anything visible to transmit it. In this way it is similar to the forces of gravity and the electric forces. Also the magnetic force between two poles changes its value in a similar manner when the distance between the poles is varied, or the strengths of the poles.

Stated mathematically, for poles concentrated sensibly at points,

$$\text{Force} = \frac{(\text{strength of pole } A) \times (\text{strength of pole } B)}{\text{distance squared}} \times K$$

where K is some constant.

This leads to a useful definition of what is to be taken as unit strength for a magnetic pole.

Unit Pole

A pole is said to have unit strength if it repels with a force of 1 dyne another exactly similar pole placed 1 cm. away from it in air.

With this definition, the constant K in the equation becomes unity, and can be left out.

A North pole without Corresponding South pole Unobtainable

If a magnetized knitting needle be broken in half, the fractured ends will be found to possess poles, one north and the other south, the strengths being about the same as that of one of the original poles. There are two magnets now instead of the one.

If the process is continued, it will be found that no matter how short the fractured pieces, they still have a north pole at one end and a south pole at the other.

Lines of Magnetic Force

Suppose, for instance, the magnetic forces near a bar magnet are investigated. Imagine a unit north pole to be placed anywhere, and consider the forces which would act on it.

The large north pole would repel the little unit pole and the large south pole would attract it, the directions of these forces being as in Fig. 20.

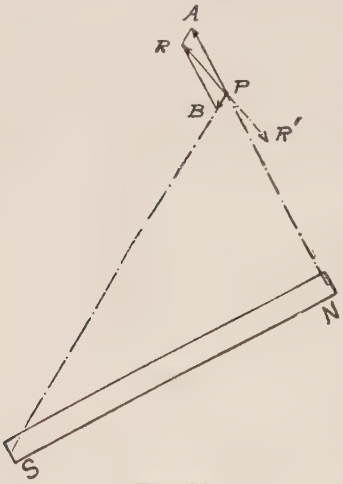

Fig. 20

Since the forces fall off inversely as the square of the distance, the relative values will be approximately as drawn, and PR gives the correct direction of the resultant magnetic force at P.

By imagining the unit north pole placed in many different positions, the direction of the resultant magnetic force for those positions could be found.

An easier method of doing this would be to employ a small compass needle. Had it been placed at P, in Fig. 20, its little north pole would have been urged in the direction PR, and its little south pole in the direction PR', because the forces on it would have the same magnitude, but opposite directions. Neglecting the effect of the earth's magnetism, the little compass needle would now set itself along the line RR', i.e. in the direction of the resultant magnetic force at P.

By placing the small compass in many different positions, a chart showing everywhere the directions of the magnetic forces could be made. (See Fig. 21.)

The lines in Fig. 21 indicate at every point along their path the direction of the resultant magnetic force, which would act on the unit north pole placed there. These lines have been drawn in one plane only; evidently they could be drawn in any plane.

Such lines are called lines of magnetic force.

"Lines of Magnetic Force" can be defined, as lines, which are drawn or imagined to be drawn, from the north pole of a magnet to the south pole, in such a manner that their direction, at every point along their path, coincides with the direction

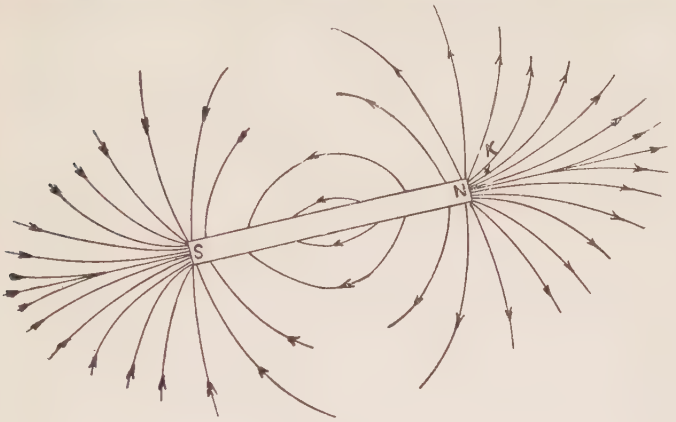

Fig. 21

of the resultant magnetic force, which would act on a unit north pole placed at the point considered.

The Lines of Magnetic Force of a Magnet do not Cut One Another

If possible, let AB and CD be parts of the lines of force of a magnet, and let them cut one another in the point O.

Then, because AB is a line of force, the direction of the resultant magnetic force at O, due to the magnet, is OR: but similarly, because CD is a line of force, it has the direction OR'. Evidently this is impossible, since there cannot be two resultant directions for the one point. Further, if a new line of force were drawn from the point K in Fig. 21, it would fall in amongst the other lines, no matter how many others were close to it.

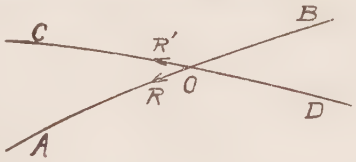

Fig. 22

If it is desired, so many lines of force can be drawn that their sides touch one another, but these lines do not cross one another.

Convention as to Number of Lines of Force to be Drawn

It has been agreed to draw, or imagine to be drawn, as many lines of force per square centimetre taken perpendicularly to the direction of the line of force, as there are dynes of force on a unit pole placed at the point considered.

The above is illustrated in Fig. 23. Here the pole-pieces of a strong electromagnet are shown.

In the space between the two pole faces a strong force

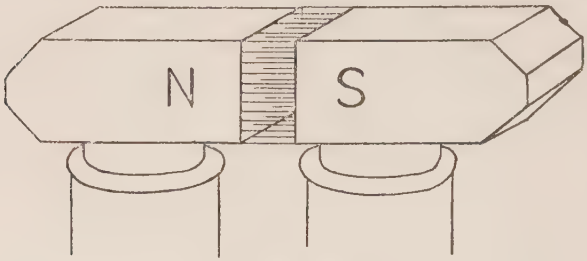

Fig. 23

would urge the unit north pole away from the north face, and a strong force would attract it towards the south face.

Evidently the magnetic forces are directed from one face to the other. Therefore the lines are to be drawn perpendicularly to the faces, except outside the gap, where they become somewhat curved.

As far as the number is concerned, if the force on the unit north pole in the gap is 3,000 or more dynes, then 3,000 or more lines per square centimetre are to be drawn.

It would not be possible or useful to draw so many lines, consequently a small number are usually drawn and the others are imagined to be there beside them.

This conception is very valuable in the calculation of values of E.M.F.s produced by conductors moving through such magnetic fields, as they are called, and also in helping one's imagination.

It is most important that the student should have a definite conception of what is meant by lines of force.

Magnetic Forces Due to a Flow of Electricity

Electricity at rest, for instance, on an insulated sphere does not influence a steel bar differently, whether it is magnetized or not.

Electricity in rapid motion does exert a force on a magnet. Supposing the charged sphere could be shot from the mouth of a cannon, and could still retain its charge whilst being projected through the air, then, as it passed, it might cause a compass needle to deflect, showing the presence of transient magnetic forces.

Electricity, in flowing along a wire, causes magnetic forces near it, because it consists really of a large number of little charges being projected through the wire. The velocity may be very great, but in the calculations of the forces produced, the velocity does not appear directly, but the rate of flow, or the number of coulombs passing per second.

Magnetic Forces Due to Current in Long Straight Wire

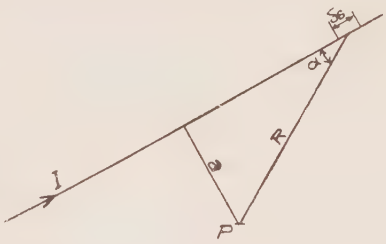

FIG. 24

Let a current of I amp. be flowing through a long straight wire, and let the magnetic forces be considered at a point P, distant a cm. from the wire.

The magnetic force at P will be found to have the value $\frac{2I}{10a}$ dynes, and the direction of this force is perpendicular to the plane containing the wire and the point considered.

Now, the total value of the force is probably the resultant of a great number of forces from each of the little charges of electricity in motion.

Supposing that the individual force at P, due to the flow of electricity in a short length, δs, of the wire, R cm. distant

from P, had the value $\frac{I \times \delta s \times \sin a}{10R^2}$ dynes, and that the line of action of this individual force at P were perpendicular to the plane containing the little piece of wire and the line joining its centre to P, then the summation of all these individual forces would give the correct total as above. Hence it can be assumed that this is the value of the individual force.

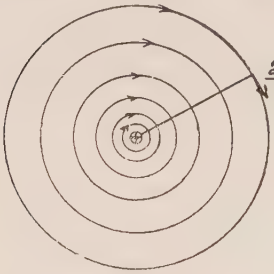

Fig. 25

Fig. 25 shows the lines of force surrounding a long straight conductor carrying a current of I amp. in a direction into the plane of the paper. These lines lie in planes perpendicular to the axis of the wire.

Evidently very great forces cannot be produced in this way without enormous currents; and as a current of 1,000 amp. would require a conductor of several square centimetres cross-section, this shows that with large currents, a cannot be very small, even at the surface of the wire.

Magnetic Forces Due to a Current in a Loop

Greater magnetic forces for lesser currents can be produced within loops of wire (see Fig. 26).

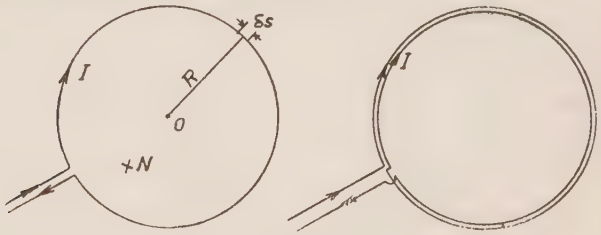

Fig. 26

Consider the forces at the centre of a circle of wire of radius R cm. All the individual forces are perpendicular to the plane of the circle, so they can be directly added.

The individual force has the value,

$$\frac{I \times \delta s \times \sin \alpha}{10R^2} \text{ dynes} \quad \therefore \text{ the total is the sum of } \delta s \left(\frac{I \times 1}{10R^2} \right)$$

The sum of δs is $2\pi R$ $\quad \therefore$ total $= \dfrac{2\pi R \times I}{10R^2} = \dfrac{2\pi I}{10R}$ dynes

The sine has the value unity, since the angle is 90°.

The force could evidently be increased by having a number of turns instead of the one. The current in each turn would help to multiply the total, but evidently, if it were attempted to have thousands of turns instead of the one, these would of necessity take up space, and the distances would be altered, in which case the force would not be multiplied in direct proportion to the number of turns.

If instead of the centre of the circle O, any other point N is considered, it will be found that electricity flowing in every part of the circumference contributes towards the total force at N.

The force for any point outside the loop is much smaller; it will be found that the individual forces, for opposite sides of the loop, oppose one another. This explains why comparatively small magnetic forces are found at the outside surface of a field-coil or solenoid.

CHAPTER IX

THEORY OF MAGNETISM

The Solenoid

THE solenoid consists of a close spiral of insulated copper wire. There may be several layers wound over the top of one another, the object being to produce great magnetic forces within the spiral or spirals.

It is important to be able to calculate the value of the magnetic forces when a given current is passed through the solenoid.

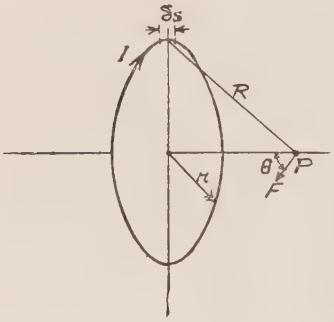

FIG. 27

The solenoid is built up of a great number of circles of wire, through which the current passes.

Consider the magnetic force produced at some point along the axis perpendicular to the plane of one of the circles, and passing through its centre, as in Fig. 27.

The value of the force at any point P can be found by summation of the individual forces, each of which is equal to

$$\frac{I\delta s \sin a}{10R^2} \text{ dynes}$$

Here sin a is equal to unity, since the angle between the little piece of wire and the line joining it to the point considered is obviously 90°.

For the individual force considered in the figure, the direction of its line of action is PF, i.e. perpendicular to the plane containing the little piece of wire and the point P considered.

Thus the individual forces at P will all have different directions, and will form a cone of forces. The resultant can be

found, by multiplying each, by the cosine of the angle between it and the axis of the cone. This is the angle θ in the diagram.

The resultant force at P will be directed along the axis, and has the value equal to the sum of $\delta s \left(\frac{I \times 1}{10R^2}\right) \cos \theta$.

But the sum of $\delta s = 2\pi r$ and $\cos \theta = \frac{r}{R}$

$$\text{so the total value of the force} = \frac{2\pi r \times I}{10R^2} \cdot \frac{r}{R}$$

$$= \frac{2\pi r^2 I}{10R^3} \text{ dynes}$$

From the above considerations, it will be seen that, anywhere along the central axis considered, the resultant force

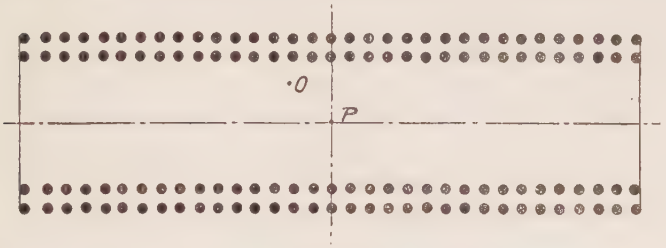

FIG. 28

is directed along the axis, and that if the force on a unit north pole is from right to left on the right-hand side of the plane of the loop, it has the same direction, i.e. from right to left on the other side of the plane of the loop.

From examination of the expression, it is evident that the force falls off rapidly with increasing distance.

From these facts, a value for the magnetic force acting along the axis of a given solenoid could be calculated.

The section of a long solenoid is shown in Fig. 28.

Consider any loop on the left of the point P. Suppose that the current has such a direction that the force on a unit N-pole at P is from right to left.

The current in every circle of wire on the left of P will contribute a small force, directed along the axis from right

to left. In each case, a second force will be provided by the turn in the corresponding position on the right of P. All the forces have the same direction and add directly.

If a summation were effected, the force at P would approach a limit, for adding to the length of the solenoid would not add much to the total value of the force at P, since R^3 becomes so large.

When the solenoid is made long enough, the limiting value of the force at P can be shown to be $0{\cdot}4\pi \times$ ampere-turns per centimetre of length, or

$$H = \frac{0{\cdot}4\pi NI}{l} \text{ dynee on unit pols}$$

where N is the total number of turns, I the current in amperes, and l the length in centimetres of the solenoid.

The above is a very important result.

Nothing has been said about the value and direction of forces for points within the solenoid, but not situated on the central axis.

It can be shown that, except near the ends of the solenoid, the forces anywhere within a long solenoid have about the same value as at the central axis, and their directions are parallel to the central axis.

Magnetization of Soft Iron

If a soft iron rod be inserted into the solenoid, it will be found to become very strongly magnetized, and attract very strongly other pieces of iron. Two pieces of iron, say, 1 in. in diameter, with flat ends, can scarcely be pulled apart by hand, whilst inside a strongly excited solenoid.

If the current be switched off, the iron will be observed to have lost nearly all the magnetism, which it had acquired in the strong magnetic field. Had a hard steel bar been used, it would have retained a large proportion of its magnetism.

Molecular Theory of Magnetism

This was due in the first instance to Weber, and has been enlarged and improved by subsequent investigators, notably by Sir Alfred Ewing. Ewing supposed all magnetic materials, such as iron, steel, nickel, and cobalt, to be composed of molecules, each of which behaved like a minute permanent magnet.

In a piece of unmagnetized soft iron, these little permanent magnets have all arranged themselves in groups, which give rise to no external magnetic forces. Possible groups are indicated in Fig. 29. The little magnetized particles are represented by a short line with an arrow at the one end. The arrow indicates the north end.

The bar of soft iron in Fig. 29 is supposed to be situated in the magnetic field inside a solenoid, the axis of the solenoid and of the bar being parallel to one another. The magnetic

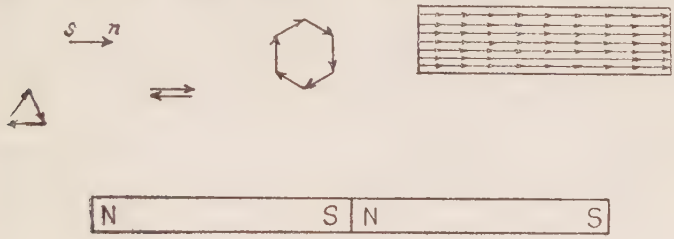

FIG. 29

force being present in the ether is thought to act also in the ether in the midst of the iron.

The little groups of molecular magnets are acted on by the magnetic force inside the solenoid, and many of the little magnets swing round and point in the direction of the magnetic force, like little compass needles. They form themselves into chains, as indicated in Fig. 29.

If two exactly similar bar magnets are placed end-on with north and south pole in contact, as in Fig. 29, little or no magnetism is now detected where the two poles are in contact, but the extreme ends have about the same pole strength as previously.

Another way of looking at the matter would be to consider the forces from the two poles in contact, which would act on a unit north pole placed at the point P in Fig. 29.

There would be a force of attraction to the south pole, and a practically equal force of repulsion from the north pole: the line of action of these two equal and opposite forces is practically

the same, and the resultant can be regarded as equal to zero. This is what is generally meant when it is stated that two equal and opposite poles very close to one another nullify each other's polarity.

Only the ends of the bar in Fig. 29 exhibit magnetic poles. Everywhere, except at the ends, all the little poles nullify each other's polarity. All the little north poles peep out of the one magnetized face, and all the little south poles out of the other.

When the iron is either taken away from the solenoid, which is supplying the magnetic field, and causing the alignment of the little molecular magnets, or when the current through the solenoid is switched off, the soft iron bar loses most of its magnetism. This is supposed to be due to the little magnets reverting to their original groupings. A little magnetism generally remains, even in soft iron, and quite a large proportion in very hard steel: permanent magnets are therefore made of hard steel.

Magnetic Saturation in Iron and Steel

Ewings's theory explains why a magnetic field of moderate strength is able to magnetize a piece of iron strongly, whereas, increasing the strength of the magnetic field manyfold, does not produce much increase in the strength of the poles of the specimen.

A moderate magnetic force has broken up most of the groups, and pulled the little magnets into line with the force, and an increase of force has only the few more stable groups to work on.

Evidently the state of magnetization produced in a given bar of iron should be measured, if possible, by the number of chains of magnetized particles produced per square centimetre taken perpendicularly to the direction of the chains.

Magnetic Lines Form Closed Curves

Consider the permanent magnet shown in Fig. 21. In the space round the magnet there are lines of force and, inside the material of the magnet, chains of magnetized particles.

The number of lines of force emanating from one pole would evidently be connected with the number of chains of magnetized particles ending on that pole face. Supposing it

were assumed that each chain of magnetized particles within the iron or steel gave rise to one line of force emanating from the north pole and stretching round to the south pole. This would form a closed path returning on itself.

It could do no harm to make such an assumption, if it is kept in mind that it is only an assumption.

The whole closed path could be termed a magnetic line. It is made up partly of a line of force and partly of a chain of magnetized particles.

The degree of magnetization of a specimen of iron or steel would be measured by the number of magnetic lines produced in it per square centimetre.

CHAPTER X

FLUX AND PERMEABILITY

Testing the Magnetic Properties of a Specimen of Iron

If a given specimen of iron or steel could be placed in a magnetic field of known strength, and the number of magnetic lines produced in it per square centimetre of cross-section could be then measured, very valuable information about the magnetic properties of the specimen could be obtained.

The solenoid at once suggests itself as a means of producing a field of known strength, since none of the quantities involved are difficult to measure. The current can be accurately measured with an ammeter, and the number of turns per centimetre of length is easily counted.

A difficulty, however, arises, since, as soon as the straight bar inside the solenoid becomes magnetized, new magnetic forces are called into play by the presence of the poles created at the ends of the bar, and the magnetic forces within the bar are now due to forces from the poles as well as from the electricity flowing in the windings of the solenoid.

The Anchor Ring

This difficulty could be got rid of if the solenoid and iron specimen were circular in form, as in Fig. 30.

In Fig. 30 a soft iron bar of circular section is shown bent into the form of a very nearly closed ring.

This ring is wound with insulated copper wire, in several layers, like the winding of a solenoid. The winding is connected through a reversing switch, ammeter, and variable resistance to a battery.

When current passes, it creates a magnetic field inside the winding, in this case amongst the iron. The winding really forms a solenoid, which is bent so that its axis becomes circular. It has been fully explained how the resultant magnetic force inside the straight solenoid, at any point, is mainly due to the electricity flowing in the near-by turns, and that

the more distant turns do not contribute much to the total force. Thus, in the case of the solenoid with circular axis, it would be reasonable to expect the forces within the solenoid to have directions following the circular axis, that is, the lines of force within the solenoid would have the form of circles, concentric with the central axis of the solenoid. This is found to be the case, and it is further found that the value of the

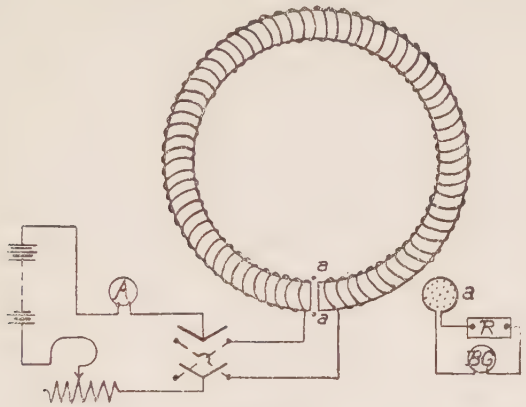

Fig. 30

magnetic force is sensibly the same as within the long straight solenoid.

It can be assumed, then, that the magnetic force, for practical purposes, has the value $\frac{0{\cdot}4\pi NI}{l}$ anywhere inside the solenoid, N being the total number of turns, I the current in amperes, and l the mean length, that is, the length of the central axis.

The ring in Fig. 30 will become magnetized, but the magnetization this time will produce two poles which are so near to one another that they do not cause a great alteration of the original magnetic force to which the iron is subjected. Further, it will be seen later that the gap in the ring can be dispensed with altogether: at present it is required, because the number of lines of force in the gap is to be measured, and from that the number of magnetic chains in the iron is to be inferred.

Method of Measurement of Number of Lines

The next problem is how to measure accurately the number of lines of force stretching from face to face across the short gap in the ring.

It is known that the cutting of lines of force by a conductor, at the rate of 100,000,000 per second, produces an E.M.F. of 1 volt in the conductor, so long as the cutting of the lines by it continues at that rate.

If, then, a wire were moved through the gap in such a manner as to cut all the lines present there, and the time and the average value of the E.M.F. could be measured, the number of the lines could be calculated.

It would not be easy to measure the E.M.F. and the short time directly, but their product is easily measured, for if the ends of the wire, which cuts the lines, are connected through a suitable resistance to a ballistic galvanometer, the deflection of the galvanometer will indicate the quantity of electricity which has suddenly passed, and the number of lines cut can be calculated.

The electrical connections are indicated on the right in Fig. 30.

Suppose that the total number of lines in the gap is N, and that the movement of the wire is such as to cut them at a uniform rate in a time t sec., then

$$\text{E.M.F.} \quad = E = \frac{N}{10^8 \times t} \text{ volts}$$

$$\text{Current} \quad = I = \frac{E}{R} = \frac{N}{10^8 \times t \times R} \text{ amp.}$$

$$\text{Quantity} = Q = I \times t = \frac{N \times t}{10^8 \times t \times R} = \frac{N}{10^8 \times R} \text{ coulombs}$$

The quantity of electricity, Q, discharged round the little circuit is seen to be independent of the time.

This merely means that if the motion is more rapid, the E.M.F. is larger, but its duration is correspondingly shorter.

Ballistic Galvanometer

No exact description of the galvanometer is given here, as galvanometers and their properties are fully discussed in books on Electricity and Magnetism.

The galvanometer used here would be of the reflecting type, the deflection being measured by the movement of a spot of light over a scale.

A suitable galvanometer consists essentially of a small coil of very fine insulated wire, suspended between the poles of a permanent magnet. When current passes through this coil, there is a force on the coil, proportional to the current, tending to turn the coil about its vertical axis. The suspension resists this movement, and the coil takes up, for steady currents, some position in which the torque due to the current is balanced by the opposing torque due to the twist of the suspension. A small mirror about the size of a threepenny piece is attached to the coil, and the movement of this mirror causes a corresponding movement of the reflected beam of light. A millionth part of an ampere may cause a full deflection.

For present purposes the galvanometer is used somewhat differently, since there is no steady current, but only one lasting a small fraction of a second. The transient current commences to deflect the moving coil, but since this is not without mass, it does not immediately lose the angular velocity acquired during the passage of the current, and the spot of light swings forward over the scale with diminishing velocity, and on coming to rest swings back through zero to the other side. The amplitude of the swings gradually lessens, but it generally takes some time for the spot to come to rest again at the zero of the scale.

The amplitude of the first swing can be shown to be proportional to the quantity of electricity which has suddenly passed. Since there is only a proportionality, the constant of the galvanometer must be found before the quantity of electricity corresponding to a given swing is known.

Determination of Galvanometer Constant

An easy way of determining the galvanometer constant is as follows (see Fig. 31).

A current of value I amp. is passed through a long solenoid of known number of turns per centimetre of length. This produces a field in the middle of values $\frac{0{\cdot}4\pi NI}{l}$ lines per sq. cm. A search coil, S, of known number of turns is placed in the

middle of the solenoid, each turn enclosing a known area and therefore enclosing a known number of lines of force.

Faraday established the fact that if a loop of wire encloses a given number of lines of force and these lines are removed, during their removal an E.M.F. acts one way or the other round the loop.

This E.M.F. invariably has a numerical value proportional to the rate of change of lines embraced by the loop, a rate of change of 100,000,000 lines per second giving 1 volt. What goes on outside the loop does not matter, only the lines enclosed are concerned. When the main current is switched off, the lines embraced by each turn of the search coil disappear. A small E.M.F. is set up in each turn, the total E.M.F. being

FIG. 31

n times that of one turn, where n is the number of search-coil turns.

Supposing that the lines embraced by the search-coil disappeared uniformly during a very short time (t sec.), then, if E is the total E.M.F. produced in the search coil, and A the area in square centimetres embraced by a turn of the search-coil,

$$E = n\left\{\frac{0{\cdot}4\pi NI}{l}\right\}\frac{A}{10^8 \times t} \text{ volts}$$

$$\text{Transient current } i = \frac{E}{R} = n\left\{\frac{0{\cdot}4\pi NI}{l}\right\}\frac{A}{10^8 \times t \times R} \text{ amp.}$$

Quantity of electricity discharged

$$= Q = i \times t = n\left\{\frac{0{\cdot}4\pi NI}{l}\right\}\frac{A}{10^8 \times R} \text{ coulombs}$$

The swing of the ballistic galvanometer is observed, and the connection between the quantity and the swing is thus established, since all the quantities in the above expression for Q are known.

Measurement of Number of Lines

The total number of lines crossing the gap in Fig. 30 can now be measured. It is evidently not necessary to move the wire through the gap in order to get the deflection on the galvanometer. If instead, a stationary loop, *aa*, in Fig. 30, embracing all the lines is used, this will have an E.M.F. in it, when the lines which it embraces are removed. The lines can be removed by switching off the current or, better still, they can be removed, and restored in the other direction by reversing the current in the windings.

Reversing the current gets over the difficulty that a piece of iron forming an almost closed magnetic path does not at once become demagnetized on the removal of the magnetizing force.

If the magnetizing current is reversed, the magnetic force is reversed, and this at once pulls the little magnets forming the chains round, and the chains are re-formed in the other direction.

A small E.M.F. will be produced in the search-coil during the removal of the lines which it embraces, and a second E.M.F. will be produced when the lines are restored in the opposite direction.

These two E.M.F.s have the same direction, and a second quantity of electricity Q, equal to the first one, is discharged round the circuit.

From the deflection observed, the total number of lines embraced by the search-coil can be calculated.

Flux, Flux-density, and Permeability

The total number of lines is called the flux. This is perhaps not a very good name, since it implies a flow. It is not definitely known what is taking place in the ether in the gap: for present purposes the magnetic forces, which are certainly there, are represented by lines of force, according to the meaning of lines of force as already explained.

The term "flux" means a bunch or a group of magnetic lines, and the term "flux-density," the number of lines per

square centimetre or per square inch, taken perpendicularly to the direction of the magnetic lines.

Flux and flux-density are equally applicable to magnetic lines within the magnetic material.

The flux-density produced in a piece of iron or steel is evidently, in the light of what has been said previously, to be taken as the standard by which the state of magnetization of the material is to be judged.

In air, the magnetic force in dynes on a unit north pole and the flux-density are, by the meaning attached to lines of force, represented by the same number. The letter H is used to denote the magnetic force on unit north pole, and B the flux-density. In air, B is equal to H.

In a magnetic material, B is in most cases much greater than H. For instance, if a piece of soft iron is placed in a field where $H = 10$, it is possible that B may have a value in the neighbourhood of 13,000. The iron being soft, the small magnetic force has been able to build a very great number of magnetic chains.

This is sometimes explained by saying that the iron is very permeable to the magnetic lines, and the *permeability*, μ, is the value of the ratio of B to H.

$$\mu = \frac{B}{H} \text{ or } B = \mu H$$

Except in the case of iron, steel, nickel, and cobalt, the permeability of all other substances does not differ greatly from unity and, for engineering purposes, may be taken as unity.

Measurement of B and H in a Specimen of Iron

The apparatus in Fig. 30 can be used. It will be found that the air-gap can be dispensed with, for if the search-coil, which in the position *aa* embraces only lines of force, is slipped over the iron so as to embrace the chains of magnetized particles and only the few lines of force which would be present if the iron were removed, it will be found that on reversal of the magnetizing current the galvanometer deflection is sensibly unchanged.

This is an important point. It is no longer necessary to have a gap in the iron, but the flux-density B, produced in the specimen, can be measured with the help of a search-coil embracing the iron.

Without the air-gap it can be said that every particle of the iron is subjected to a magnetic force of sensibly the same value, that value being known from the number of turns, the mean length of the circular solenoid, and the current flowing.

By adjusting the current to a series of suitable values, reversing it, and reading the throw of the calibrated ballistic galvanometer, the values of B for a number of values of H can be obtained. If these are plotted as shown in Fig. 32,

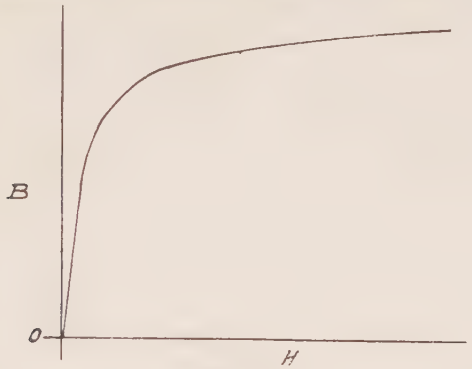

Fig. 32

what is called a B–H or magnetization curve for this quality of iron is obtained.

With soft iron flux-densities, up to about 23,000 lines per square centimetre are obtainable with H in the neighbourhood of 1,200 dynes on unit pole. A flux-density of 15,000 could have been obtained with a value of H of only 23. The curve still mounts slowly, however great the flux-density; but after it has become so flat, any considerable increase in the value of B is only obtained at the expense of a great increase in the value of H.

Curves of the permeability could be drawn if desired, but curves of the B–H type just discussed are really more useful to the engineer.

The permeability is a very variable quantity. Not only is it widely different for different magnetic materials, but also it is different for every different value of flux-density in the one material.

CHAPTER XI

THEORY OF MAGNETISM (*contd.*)

The Magnetic Circuit

THE iron ring in Fig. 30 constitutes a complete magnetic path or circuit, since starting anywhere in the iron and following along the chains or magnetic lines leads back to the starting point. At any cross-section taken perpendicular to the lines, the same total flux would be present.

In this case the magnetizing windings are distributed along the whole length of the magnetic path; and since in machines the magnetizing windings must generally be bunched together into one or two coils, extending only over part of the magnetic path, this case ought to be investigated.

The apparatus depicted in Fig. 30 can still be used if the magnetizing coils are re-wound as indicated in Fig. 33, the same total number of turns being wound as before.

The search-coil is placed in the position *AA*, and afterwards *BB*. The current is adjusted to one of the values used in the last experiment, and is then reversed, the galvanometer swing being noted.

Readings for several values of the current should be taken, and the results compared with those previously botained. It would be found that the flux present at *AA* was somewhat in excess of that at *BB*, the one being a little greater and the other a little less than was obtained before for the same current.

This result is of extreme importance. Almost the same flux is present right round the magnetic circuit as before.

In the previous experiment, every part of the iron was directly subjected to the magnetic force from the flow of electricity through the distributed winding. Now only two portions of the circumference of the ring come under the action of such a force, and the force has a greatly increased value there, and yet the iron is not found to be very much more strongly magnetized inside the coil. What is the explanation?

Suppose the iron to be cut through at the four places marked *AA* and the unwound parts removed. The pieces of iron left inside the coils would not become so strongly magnetized for a given current as might at first be supposed, because the poles produced on the end surfaces would exert magnetic forces at any point within the winding opposing the

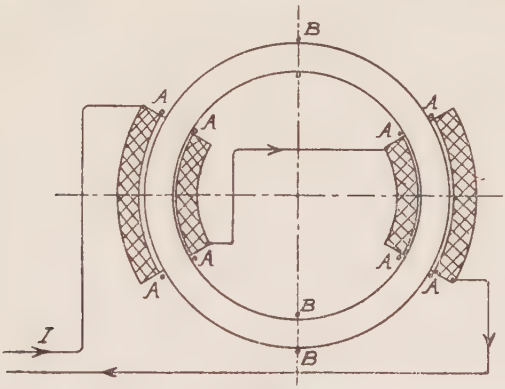

FIG. 33

magnetic force set up by the current in the windings, and the resultant magnetic force inside the solenoid would be lessened.

If the current had such a direction as to make the upper left-hand surface of iron at *AA* north, then the upper right-hand surface would be magnetized south. If the piece of iron which has been removed were replaced between these surfaces, it would become magnetized as does every piece of iron placed in the magnetic field between a north and south pole.

Every chain which was formed within this bridge piece would lessen the free magnetism at *AA*, and the flux in that part of the iron within the coils would be increased.

The better the magnetic path through the bridge piece, the greater the flux throughout.

Replacing the bottom bridge piece has the same effect.

The above explanation is incomplete, but the important fact remains that if pieces of iron are put together to form roughly a ring or square, then if magnetizing windings encircle some part or parts of the ring or square, the magnetic forces

produced there magnetize the whole ring or square right round, and the flux throughout has roughly the same value.

This enables one to speak of the above as a magnetic circuit, and it is of great importance to be able to calculate what flux will be produced in a given magnetic circuit, by a given number of magnetizing turns, carrying a given current.

Rules for Finding Values of Flux in a Magnetic Circuit

Experimental work has led to rules for predicting the value of the flux to be expected under given conditions. The magnetizing effect of a given current which encircles part of the magnetic path, i.e. the path taken by the magnetic lines, is proportional to the value of the product of the current and the number of turns. This product is often called the ampere-turns, written AT.

Reluctance

The term " reluctance " indicates that there is difficulty in creating a flux throughout a piece of magnetic or other material.

It can be easily shown experimentally that, keeping the flux-density constant, the number of ampere-turns required to produce a given flux in a given quality of material, depends directly on the length: if the area of cross-section is doubled, this doubles the flux without alteration of the number of AT.

If the length and cross-section are kept constant, the number of AT required for a given flux is inversely proportional to the permeability, μ, of the material at that flux-density.

$$\therefore \text{Reluctance} \propto \frac{\text{length of magnetic path}}{\text{permeability} \times \text{cross-section of path}} \propto \frac{l}{\mu \times A}$$

The flux is found to be directly proportional to the AT if the reluctance is kept constant, and inversely proportional to the reluctance if the AT are kept constant, so an equation of the following form can be set up.

$$\text{Flux} = \frac{AT}{\text{reluctance}} \times \text{some constant.}$$

Equation for Value of Flux

If it is decided to take the reluctance as being numerically equal to the length of the magnetic path in centimetres,

divided by the product of the permeability and the area of cross-section of the magnetic path in square centimetres, the constant has the value $0{\cdot}4\pi$ or $1{\cdot}257$, then

$$\text{Flux} = \frac{1{\cdot}257 \times AT}{\text{reluctance}}$$

In applying this rule there are many points which must be carefully noted.

It is only applicable to a circuit, which is truly the path taken by the lines. Sometimes, especially inside machines, it is quite difficult to decide the exact path which will be taken by the lines.

Flux is taken to mean either the entire number of lines present at some section taken perpendicular to their path, or it can be taken to mean a bunch of lines forming part of the total flux, in which case this bunch must be followed right round and the reluctance of its part of the path taken.

To illustrate this point, the flux in the ring in Fig. 30 may be considered. In the sense of the rule, either the whole flux may be considered, or one might consider a bunch of lines, say, those grouped round the circular centre line of the iron and passing through a square centimetre of the cross-section. This bunch would then have to be followed right round, and the reluctance of the path would depend partly on whether these lines spread out farther from each other or, contracted, to occupy less cross-sectional area at some point or points along their path.

Magnetic Circuit in Dynamo

Another important point is to remember that, for the purpose of applying the rule, AT really mean amperes enclosed by the path of the flux. This important point is illustrated with reference to Fig. 34, in which the magnetic circuit of a D.C. machine is depicted.

Considering the group of lines of which AA is the central one, the amperes enclosed are those in section (b) and section (c) of the shunt coils. These amperes and no others are enclosed by the path of the lines considered. The amperes in the sections (a) and (d) are not enclosed by the group of lines under consideration.

This sometimes confuses the student, but he should remember that he is merely applying a rule, and must be careful to do this only in the way which was intended.

He sometimes asks: Why any amperes near, but not actually enclosed, are not to be reckoned as AT for the flux considered, and do these have no effect? Of course, they do have an effect. If they were not present, the path taken by the bunch of lines under consideration might have been quite different. All the currents flowing near enough must be taken into account in deciding what is really going to be the path of any

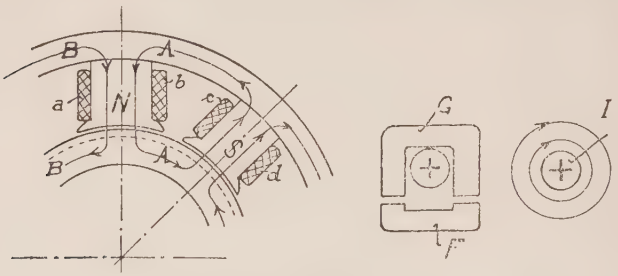

FIG. 34

bunch of magnetic lines, but once the path of any one bunch has been correctly determined, then only the amperes actually enclosed by those lines are to be used in the equation.

Special Magnetic Circuit

In Fig. 34, on the right, a little magnetic circuit is depicted to emphasize that it is amperes enclosed, which really count, and that, when AT are spoken of, it is really another way of saying amperes enclosed, since, for example, the number of amperes in the cross-section of a shunt field coil in Fig. 34 is found by multiplying the current by the number of turns in the coil. In the little magnetic circuit referred to there are no turns: the current flows straight through.

F and G are packets of iron plates, built up to a thickness of perhaps half an inch. The current causes the iron to become magnetized, and the iron part F, which is moveable, is attracted to the other part G. Such an arrangement is often used to trip the catch of a circuit-breaker or automatic switch.

When the current reaches a certain value, the part F snaps up against the part G and knocks out the catch.

In the equation for this little magnetic circuit there are no turns.

Magneto Motive Force

The quantity $1{\cdot}257 \times$ the amperes enclosed is called the magneto motive force, M.M.F. It has a significance, dealt with in books on Electricity and Magnetism, for it is equal to the work in ergs, done in carrying a unit north pole, by any closed path, once round the current, against the magnetic forces.

If I, in Fig. 34, represents a conductor carrying a current I amp. in a direction into the plane of the paper, the magnetic forces will be indicated by the lines of force shown, and the value of the magnetic force, at a distance a cm. from the centre of the conductor, will be $\frac{2I}{10a}$ dynes on a unit pole. If the pole be carried round the circle once, this force will be exerted through a distance $2\pi a$ cm., and the work done will be $\frac{2I}{10a} \times 2\pi a = 0{\cdot}4\pi I$ ergs, and equals $1{\cdot}257$ times the number of amperes enclosed.

Reluctances in Series

The reluctance of the magnetic circuit is to be taken as the sum of the reluctances of the various parts making up the entire path. This is similar to the addition of resistances in series.

The formula for the flux then becomes

$$\phi = \text{Flux} = \frac{\text{M.M.F.}}{\text{reluctance}} = \frac{1{\cdot}257 \times AT}{\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3} + \text{etc.}\right)}$$

where l_1 is the length in centimetres of the mean magnetic line through part 1, a_1 the cross-section of the path there, and μ_1 the permeability for that flux-density in the material of which part 1 is composed. The circuit must be divided up into parts, each time there is a change of cross-sectional area or of material.

Practical Methods

The engineer often uses the above result in a somewhat different form, for if in the above equation one cross-multiplies, the result is—

$$1{\cdot}257 \times AT = \phi \frac{l_1}{a_1\mu_1} + \phi \frac{l_2}{a_2\mu_2} + \phi \frac{l_3}{a_3\mu_3} + \text{etc.}$$

$$= \frac{B_1 l_1}{\mu_1} + \frac{B_2 l_2}{\mu_2} + \frac{B_3 l_3}{\mu_3} + \text{etc.}$$

since the flux divided by the area gives the flux-density B.

Evidently the total AT, on the left-hand side, is made up of the sum of the numbers of AT for each of the parts, 1, 2, 3, etc., of the circuit.

It is possible, then, to proceed somewhat differently, if more convenient, in finding the total AT for a given flux through a given magnetic circuit.

Assume the flux desired to be present, and find the AT for each portion.

If the above equation were multiplied throughout by 0·8, the 1·257 × 0·8 would become practically unity.

The AT required on account of portion 1 of the circuit would be $\frac{0{\cdot}8B_1l_1}{\mu_1}$ or $\frac{0{\cdot}8B_1}{\mu_1}$ per centimetre of length of path.

The engineer supplies himself with curves, found experimentally, for each quality of material, in which B is plotted against the value of AT per centimetre of path, as in Fig. 35. He then supposes the required flux to be present and finds the value of B, which would be caused in each part.

Reference to the curves shows how many AT per centimetre would be required, and multiplying this by the length of the mean magnetic line through the part concerned, shows him how many AT will be required for this part. In the same manner he finds the AT for all the other parts, and the sum gives the total AT, which must be supplied to produce the flux, which has been supposed to be present.

For instance, supposing that in Fig. 34 it had been desired to have an average flux-density in the air-gap of 9,000 lines per square centimetre, the flux per pole would have been equal to 9,000 times the area of the pole face. The area of the pole face would be taken slightly larger than its actual

measurements indicate, to allow for the fringing of the lines at the edges.

The path taken by the lines in the gap is mostly towards the tops of the teeth, and the flux-density over the teeth is greater than the average value, needing more AT, as is explained later under "Carter's Air-gap Coefficient." Supposing that in the present case this had had the value 1·1, then the

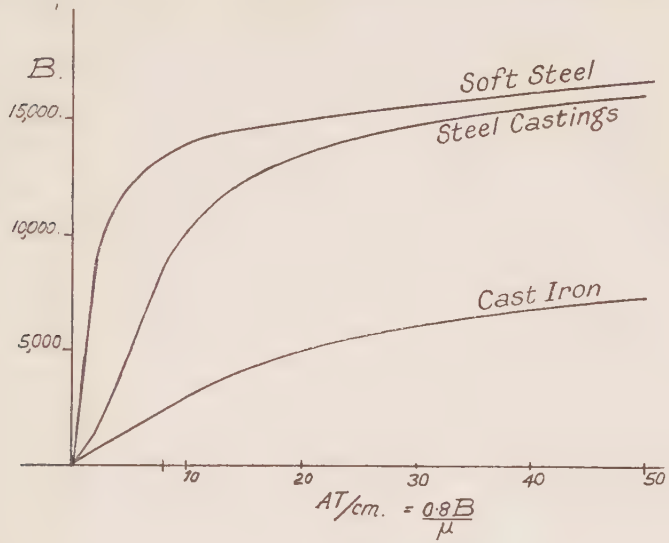

FIG. 35

AT required for the gap will be 1·1 times greater, on account of the armature being toothed instead of smooth.

Suppose the air-gap to be 0·5 cm.,

$$\text{the necessary } AT = 0{\cdot}5 \times AT \text{ per cm.} = \frac{0{\cdot}5 \times {\cdot}8B}{\mu}$$

$$= \frac{0{\cdot}5 \times {\cdot}8 \times 9000 \times 1{\cdot}1}{1} = 3960\ AT$$

Remember that the AT per centimetre required for any part of any magnetic circuit is equal to $\frac{0{\cdot}8B}{\mu}$. Wherever the path

is through air or any non-magnetic material, μ can be taken as unity, and the AT per centimetre can be found without the help of curves.

Carter's Air-gap Correction Coefficient

It is really only necessary to consider the part from the centre line of one slot to the centre line of the adjoining tooth. Carter's coefficient is the ratio of the maximum flux-density in the air-gap to the average. The average flux-density is assumed to be known: it is one of the quantities often assumed at the outset of the design of a machine. For large D.C.

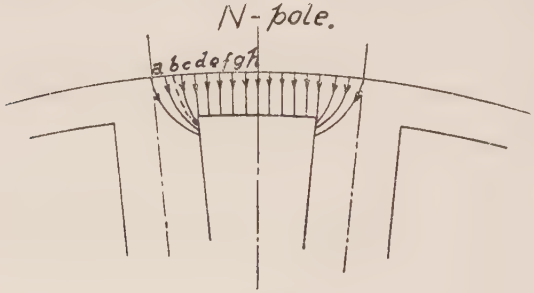

FIG. 36

machines it may have a value between 9,000 and 10,000 lines per square centimetre.

Only relative numbers are required here, since it is a ratio which is to be evaluated.

Make a sketch, to an enlarged scale, of the part indicated in Fig. 36, dividing up the half tooth-pitch at the pole surface into any convenient number of equal parts.

From each point of division draw a line of force as one considers it would be. Consider a depth of, say, 1 cm. behind the plane of the paper. The lines emanating from the area of the pole under consideration are thus divided up into a number of tubes, all having equal bases. Remember that a tube of lines of force is any bundle of lines of force bounded on all its lateral surfaces by lines of force; it contains the same number of lines throughout its length. The tubes are all in parallel with one another, and the relative numbers of

lines in each is inversely proportional to the reluctance. Hence figures for the relative or actual reluctances are required.

For convenience, call the flux in the tubes *d*, *e*, *f*, *g*, *h* 1,000 lines.

Next with this number find the number of lines in the tubes *a*, *b*, *c*. Evidently the reluctance of these latter tubes must be compared with that of the tubes *d*, *e*, *f*, *g*, *h*.

To obtain the relative reluctance of, say, tube *a*, find, with the dividers, the mean length of path on the sketch. The mean width must now be found by taking a number of widths at equal distances apart all along the tube, then

$$\frac{\text{Reluctance of tube } a}{\text{Reluctance of tube } f} = \frac{\text{mean length of path of } a}{\text{mean width of path of } a} \div \frac{\text{length of path of } f}{\text{width of path of } f}$$

Supposing that this, on being worked out, showed a value 1·5, then tube *a* has 1·5 times the reluctance of a tube directly over the tooth tip, and the flux would consequently be $\frac{1000}{1{\cdot}5} = 666{\cdot}6$.

Supposing tubes *b* and *c* had 750 and 833·3 lines in them, the number being worked out as with tube *a*. The average flux per tube would be equal to 666·6 + 750 + 833·3 + 1000 + 1000 + 1000 + 1000 + 1000 divided by 8, the number of tubes. This gives 906·25. The maximum flux was 1,000, so the coefficient has the value 1,000 divided by 906·25, or about 1·1.

PART III
ALTERNATING CURRENTS

CHAPTER XII

VECTOR DIAGRAMS

Wave-shape

It is assumed that a description of the alternator is unnecessary. The E.M.F. produced in a given group of stator conductors will reach its maximum value when the centre of a rotor pole is moving past the middle of this group of conductors, and falls off to zero as the pole moves away, to grow up to a maximum in the opposite direction as the centre of the next pole, which is of the opposite kind, approaches the centre of the group of conductors. The E.M.F. then dies away to zero and commences to rise in value in the original direction, and the cycle is repeated.

The number of complete cycles per second is called the frequency. In Europe the most usual frequency for ordinary engineering purposes is 50 cycles per second. In wireless work the frequency has very much higher values, and may be a million or more cycles per second. This latter fact should prevent any student from finding difficulty in imagining the rapid changes which take place in the values of E.M.F.s and currents, even with 50 cycles.

If the instantaneous values of an E.M.F. or current are plotted against time, the shape of graph so obtained is referred to as the wave-shape of the E.M.F. or current. The sine wave-shape is generally looked upon as being the most desirable, and the alternator is generally designed to give as nearly as possible this wave-shape, which is the same as that of a curve of sines plotted against angles.

Sine wave-shapes will be assumed for the present, the irregular wave-shapes being dealt with later.

Root-mean-square Value

In Fig. 37 the instantaneous values of a current, which reaches the maximum value of 100 amp., are plotted. The heating of value this current in passing through a resistance would be the same as that of a direct current having the steady value, 70·7 amp. The rate of the production of heat in the given resistance depends on the square of the value of the current from instant to instant, so obviously it is the mean value of the squares of the instantaneous values on which the heating value of the current depends. With the

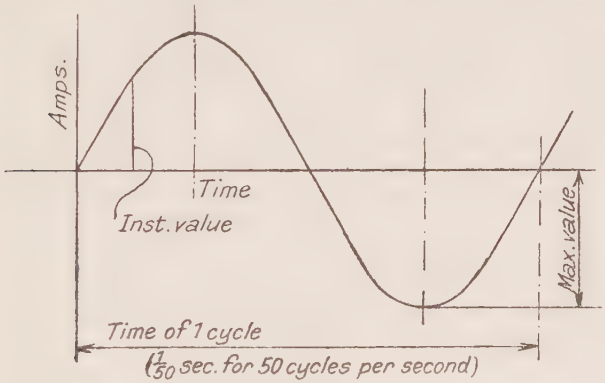

FIG. 37

current of maximum value 100 amp., the mean square of the instantaneous values is 5,000.

The direct current of 70·7 amp. steady value also has the mean-square value of 5,000. The square-root of the mean-square value of both is 70·7.

There is another advantage in calling an alternating current or E.M.F. by its root-mean-square value, since ammeters and voltmeters can be readily constructed to read this value. When one speaks therefore of a current of 250 amp., or a voltage of 6,600 volts, it is always the R.M.S. value which is meant. Had the instantaneous value, the average or the maximum value, been intended, it would have been necessary to state this.

If the maximum value is known and the R.M.S. value is

wanted, multiply by 0·707 or divide by $\sqrt{2}$. Both give the same result.

Inductance

The inductance of a coil is its property of inducing an E.M.F. in its windings, when the value of the current changes in any manner. Suppose a solenoid, without iron in it, were considered. A current sets up a flux in the core of the solenoid. The value of the flux is proportional, at every instant, to the value of the current at that instant, and any change in the value of the current produces instantaneously a proportionate change in the value of the flux.

As the flux changes, it inevitably produces a E.M.F. in each turn of the coil. The value of the E.M.F. is not dependent on the value of the flux, but on the value of the RATE of CHANGE of this flux.

Practical Unit of Inductance

If the current in a given coil changes its value at the rate of 1 amp. per second, and if the value of the E.M.F. produced in the coil happens to be exactly 1 volt, then the coil is said to possess unit inductance, or 1 henry inductance. Number of henries denoted by L.

Very great rates of change of currents are met with, without the current itself reaching any very large value.

A very usual value for the current in a D.C. generator, per armature path, is 150 amp. During the time of commutation, which may last about a thousandth part of a second, the current in an armature turn is reversed, giving a change of twice 150 amp. in a thousandth part of a second, or a rate of change of 300,000 amp. per second, without the current itself being any greater than 150 amp.

In designing the machine, care is taken that the inductance of an armature turn shall be small, possibly of the order of a hundred-thousandth part of a henry, and thus the E.M.F. produced in the coil is small, in the present case 3 volts, that is, the rate of change of current in amperes per second, multiplied by the value of the inductance in henries.

With the current in Fig. 37, the average rate of change is 20,000 amp. per second, since the current changes from zero

to 100 amp. in a quarter of a cycle, or in a quarter of a fiftieth part of a second.

The above is the average rate of change of the current. The instantaneous rate of change varies from zero, at the instant when the current is passing through its maximum value to the value 31,416 amp. per second, at the instant when the current is passing through zero. The calculation of this latter value will be understood from what follows.

Vector Diagram

The instantaneous values which voltages and currents assume simultaneously in a given circuit can be easily represented on what is known as a vector diagram.

Suppose that one had an electric motor which could be caused to rotate exactly at some desired speed. If a small hole were drilled in the shaft at right angles to the axis, to take a thin rod, pointed at the one end remote from the shaft, then, as the shaft revolved, the pointed end would describe a circle. If the length were made 10 in. from the tip to the centre of the shaft, and were taken to represent the maximum value of the current in Fig. 37, then as the pointer passed through all the positions, when revolving at 50 r.p.s. the vertical height or depth of the tip of the pointer, above or below the horizontal plane containing the axis of the shaft, would give at each instant the instantaneous value of the current to scale. Since the pointer was made 10 in. in length, the scale is evidently 10 amp. to the inch.

Since this pointer revolves at 3,000 r.p.m., it would not be possible to measure the heights and depths, but the pointer can be imagined to be revolving at full speed through any position in which it is drawn on the paper, and the instantaneous values can be scaled off for such positions. When a height is registered, the current is in the opposite direction to that when a depth is shown. The heights and depths are, of course, proportional to the sine of the angle turned through by the pointer from its horizontal position.

For a given circuit, a number of such pointers can be thought of as attached to the same shaft in their correct relative positions and, as the diagram revolves, simultaneous values are obtained for the current and a number of voltages in the circuit.

Such a diagram is called a vector diagram, and its importance in A.C. work is very great.

The Maximum Rate of Change of a Sine Current or E.M.F.

This is evidently of great importance, since the value of the E.M.F. of self-inductance depends on the rate of change of the current, and the current flowing, at any instant, into or out of a condenser, depends on the rate of change of the volts supplied to its terminals.

Two methods of finding the required value are given, the first being intended for students without much mathematical training.

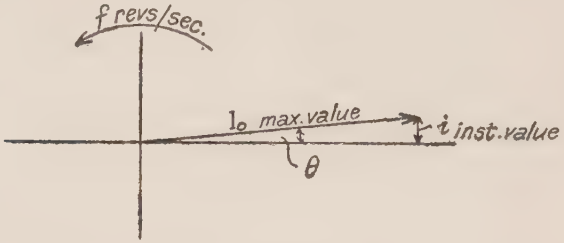

Fig. 38

I_o represents to some chosen scale the maximum value of the current concerned. Since the rate of change of the instantaneous values of the current is greatest, when the current is passing through its zero value, the vector has been drawn near that position in Fig. 38. In turning through the small angle θ, the current has grown from zero to the value i amp. The change of the current in the time taken to move through the angle θ is i amp. To find the rate of change, all that is necessary is to find this time in seconds, and to divide the value i by the time. Evidently the time for one complete revolution of the vector is $\frac{1}{f}$ sec., where f is the frequency, for f revolutions must be completed in the second. If the frequency is 50, only one-fiftieth part of a second is available for each revolution of the vector. The time for the small angle θ will be that same fraction of the time for one whole revolution, as i is of the entire circumference described by

the tip of the vector. This fraction can be seen to have the value $\frac{i}{2\pi I_o}$, so the time is $\frac{1}{f} \times \frac{i}{2\pi I_o}$ sec., and since rate of change, is change divided by the time taken for the change, the rate of change is $\frac{i}{\left(\frac{i}{2\pi f I_o}\right)} = 2\pi f I_o$ amp. per sec.

If this current flowed through a coil having $\frac{1}{8}$ of a henry inductance, the maximum value of the E.M.F. induced by the changes of the flux linked with the coil would have the value $\frac{1}{8} \times 2\pi f I_o$ volts.

In general, the maximum value of the E.M.F. of self-inductance is equal to $2\pi f L I_o$ volts, where L is the value of the inductance in henries.

EXAMPLE. Find the maximum value of the E.M.F. of self-inductance produced, when a current of 30 amp. flows through a coil of inductance 0·3 henries, the frequency being 50.

$$E_o = 2\pi f L I_o = 2\pi \times 50 \times 0{\cdot}3 \times \sqrt{2} \times 30 = 3{,}990 \text{ volts}$$

This is a large number of volts, but 0·3 henries is a large inductance to be traversed by a current of 30 amp.

Second Method. The equation for the instantaneous value of the current is evidently

$$i = I_o \sin 2\pi f t \text{ amp.}$$

where 2π is the angle in radians, turned through by the vector per revolution, and f is the number of revolutions per second, t being the time concerned, in seconds, since the vector was in the horizontal position from which the angle is to be measured. The angle would be expressed as a large number of radians, if t had more than a small value. This is of no consequence, since in determining the position of the vector all the whole revolutions can be discarded and the fraction left over used to fix the position of the vector. The rate of change of the current is equal at any instant to the value of the differential of i with respect to t,

$$\frac{di}{dt} = \frac{d(I_o \sin 2\pi f t)}{dt} = 2\pi f I_o \cos 2\pi f t$$

That is, the maximum rate of change of the current is equal to $2\pi f I_o$ amp. per sec. as before.

Vector Representing the E.M.F. of Self-inductance

Evidently the E.M.F. of self-inductance can be represented, on a vector diagram, together with the current concerned, since it has been seen that the E.M.F. follows a cosine law, that is, has the same wave-shape as the sine current, but is displaced from it by 90°. The maximum value is equal to $2\pi fLI_0$ volts, so after a scale for volts has been chosen, the length of the vector is known. Its position is 90° behind that of the current vector in the direction of rotation, which should

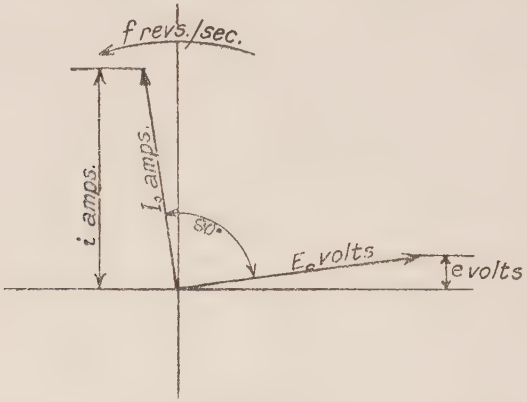

Fig. 39

in every case be marked on the diagram. The rotation should be in a counter-clockwise direction.

Lenz's Law can be made use of to assure oneself of the correct position of the E.M.F. vector relative to the current vector. It states that the E.M.F. of self-inductance opposes the current whilst the current is increasing, and assists the current whilst it is decreasing.

In Fig. 39 the current vector has just passed the position of maximum positive value, and the current is decreasing, therefore the E.M.F. of self-inductance must also have a positive value: this agrees with the diagram.

Had the E.M.F. vector been drawn 90° in front of the current vector, it would have resulted in there being a negative value at this instant, and would have been incorrect.

Vector Diagram for Circuit Containing Resistance and Inductance

An alternator giving a sine E.M.F. of maximum value E_o volts is connected to a non-inductive resistance of value R ohms in series with a resistance-less coil of inductance of value L henries.

Neglecting the resistance and inductance of the alternator and connections, find the value of the current.

The working is easier if it is assumed that a current of maximum value I_o amp. is flowing, and the value of the alternator volts to give this current is then to be found.

Kirchoff's Law can be used to find the relationship between the quantities involved. There are 2 E.M.F.s in the circuit, the one generated in the alternator and the other induced in the turns of the coil, whose inductance is L henries.

In applying Kirchoff's Law, no account is to be taken of the cause of an E.M.F. An E.M.F. of 10 volts is an E.M.F. of 10 volts, irrespective of its origin.

At any instant, then, the algebraical sum of these two E.M.F.s must be equal to the sum of the IR products at that same instant.

The algebraical sum of two or more E.M.F.s of known maximum value, and of the same frequency, can be found very easily with the help of a vector diagram.

Vectorial Addition

Suppose that in Fig. 40 the vectors A and B represent two E.M.F.s in their correct relationship to one another, both having the same frequency, then $(a + b)$ represents the value of their algebraical sum at the instant considered. If now a parallelogram is drawn, having A and B as adjacent sides, then as this parallelogram rotates about its corner at O, the height c of the tip of the diagonal will always be equal to the algebraical sum of the heights a and b. This is self-evident, when it is noticed that b and d are equal in the first position, and a and d in the second.

The diagonal of the parallelogram is itself, then, a new vector which, as it rotates, gives always a height or depth exactly equal to the algebraical sum of the heights or depths of the two original vectors.

This result is of great importance. It is called vectorial addition, and can be extended to any number of vectors, any two being added first, and the diagonal obtained then combined with any one of the remaining vectors, and so on, until one is left with a single vector, which, as it rotates, gives a

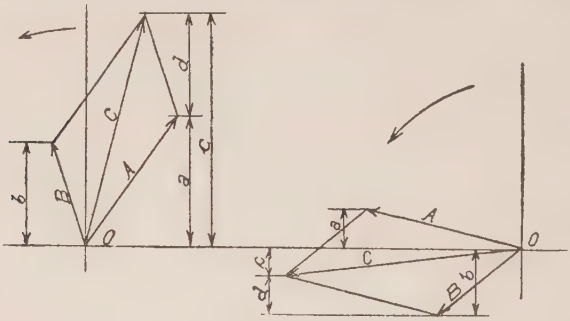

FIG. 40

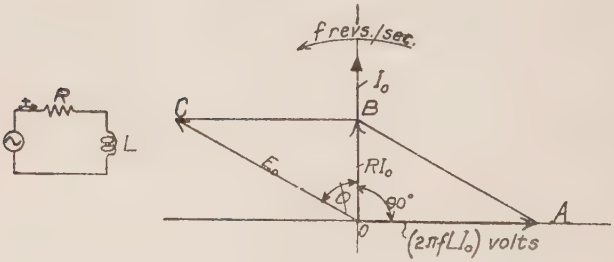

FIG. 41

height or depth equal to the algebraical sum of all the heights and depths.

Vector Diagram (*contd.*)

The vector diagram in Fig. 41 will now be quite easily understood. The vector I_o represents the maximum value of the current to some suitable scale. It can be drawn in any position. Next, the vector for the E.M.F. of self-inductance is inserted 90° behind the current vector. The length is made to represent the maximum value, $2\pi fLI_o$, to some scale for volts. There are several volt vectors and they must be all

drawn to the same scale. The second volt vector can now be inserted, having a length equal to RI_o to the scale of volts.

This is intended to represent with its height and depth, as it rotates, the value of the IR product, complete with sign. The sign is always that of the current at the instant, so the vector must evidently coincide in position with the current vector. In the diagram it is drawn shorter than the current vector, but this does not necessarily mean that the resistance has a value less than unity, because I_o is drawn to the ampere scale, whereas I_oR is drawn to the volt scale.

The next step is to complete the parallelogram, of which I_oR is the diagonal and $2\pi fLI_o$ is one side. This will be found to fix the length and angular position of the alternator E.M.F. vector. Suppose E_o to represent correctly the maximum value of this latter E.M.F., then, as the diagram rotates, the heights and depths of the tip of the diagonal OB will give the algebraical sum of the two E.M.F.s to scale, but these heights and depths are also equal to the sum of the IR products to the same scale. Hence with this value of the alternator E.M.F., Kirchoff's Law is satisfied at every instant. No other E.M.F. could satisfy the requirements.

In the diagram, OBC is a right angle and CB is equal to OA. Suppose that the scale of volts happened to be 1 mm. to the volt, then E_o^2 would be the number of square millimetres in the square on OC, but this is known to be equal to the sum of the areas of the squares on OB and BC, and these numbers of square millimetres are $(RI_o)^2$ and $(2\pi fLI_o)^2$, and hence the following equation between these numbers must be true—

$$E_o^2 = (RI_o)^2 + (2\pi fLI_o)^2$$
$$= I_o^2\{R^2 + (2\pi fL)^2\}$$

$\therefore$ transposing and taking the square root of both sides,

$$I_o = \frac{E_o}{\sqrt{R^2 + (2\pi fL)^2}}$$

This equation shows the relationship for maximum values. If both sides are multiplied by 0·707, then $0{\cdot}707I_o$ and $0{\cdot}707E_o$ are equal to the R.M.S. values of current and alternator E.M.F., or

$$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}}$$

Impedance

The expression by which the value of the E.M.F. must be divided to obtain the value of the current is called the IMPEDANCE.

The practical unit for it is the ohm, so the value of the current is obtained by dividing the number of volts by the number of ohms, as was done in applying Ohm's Law for direct currents.

The letter Z is used to denote the value of an impedance, then,

$$I = \frac{\text{volts}}{\text{impedance}} = \frac{V}{Z} \text{ or } Z = \frac{\text{volts}}{\text{amps.}}$$

Angle of Lag

It will be noticed that in Fig. 41 the current vector lags behind the alternator E.M.F. vector by an angle marked ϕ in the diagram, that is, the current reaches its maximum value after the alternator E.M.F. has passed through its maximum value. The current, of course, is proportional at any instant to the algebraical sum of the two E.M.F.s, so the current could not be expected to be in time with only one of the E.M.F.s.

The determination of the value of this angle is of considerable importance. From the geometry of the diagram, it will be seen that the tangent of the angle ϕ has the value

$$\frac{2\pi fLI}{RI} \text{ or } \frac{2\pi fL}{R}$$

Reactance

If in the circuit considered in Fig. 41 the resistance had had practically zero value, it is, of course, impossible for it to be quite zero, then the value of the impedance would have been very nearly $2\pi fL$ ohms. This quantity is called the reactance, and it is usually denoted by the letter X. It will be noticed that the value of the reactance depends on the frequency, as well as on the inductance. If it is stated that a circuit has a certain number of ohms impedance or reactance, the frequency for which it has these values should at the same time be stated. The inductance is not dependent on the frequency.

It will be noticed that the angle of lag becomes virtually

90°, when the inductance is considerable and the resistance is made as small as possible, and that the angle of lag is almost zero if the resistance is considerable and the inductance is made as small as is practicable.

Numerical Examples

EXAMPLE. Find the value of the current which will flow from an alternator giving 250 volts at 50 cycles through a resistance of 12 ohms, connected in series with an inductance of 0·127 henries.

$$Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{(12)^2 + (2\pi\ 50\ 0{\cdot}127)^2}$$
$$= \sqrt{(12)^2 + (40)^2} = \sqrt{1744} = 41{\cdot}7 \text{ ohms}$$
$$\therefore\ I = \frac{250}{41{\cdot}7} = 6 \text{ amp.}$$

EXAMPLE. Find the value of the current which will pass through a coil having a negligible resistance and 0·3 henries inductance, when 600 volts at 60 cycles are applied to its terminals.

$$I = \frac{600 \times 10}{2\pi 60 \times 3} = \frac{50}{3\pi} = 5{\cdot}3 \text{ amp. (nearly)}$$

Power

The instantaneous value of the power given out by an alternator is obtained by multiplying the instantaneous value of the volts by the instantaneous value of the amperes. This value would bear a negative sign, if the current and volts had different signs, at the instant.

Evidently, if the current and the volts are out of time with one another, as in the diagram in Fig. 41, the one will reverse in sign before the other, and in the meantime their product will have a negative sign, meaning that the alternator is receiving instead of giving out energy. The greater the angle of phase difference, as it is called, between the vectors of current and volts, the less the average value of the power during a cycle. It is the average value of the power that is generally wanted, in order that it may be multiplied by the time, to give the total amount of energy supplied or received in the time under consideration.

The curves of current and volts for a supposed case are

shown plotted in Fig. 42. The curve showing the instantaneous values of the power has been obtained by multiplying the corresponding ordinates of current and volts.

If the average value of the power were obtained graphically from the curve, it would always be found to amount to $VI \cos \phi$ watts.

That it has this value could be proved mathematically as follows—

The instantaneous value of the power is denoted by p,

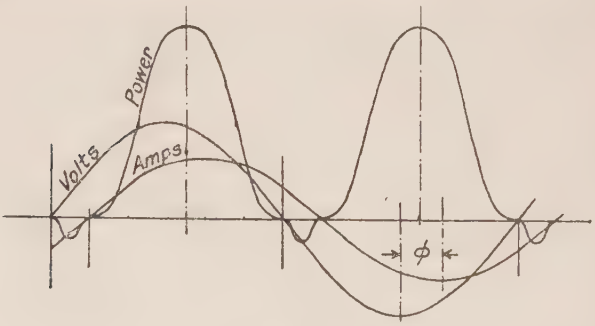

Fig. 42

and $2\pi f$, the angular velocity of the vectors in radians per second, by ω.

$$
\begin{aligned}
p &= vi \\
&= V_o \sin \omega t \times I_o \sin (\omega t - \phi) \\
&= V_o I_o \sin \omega t (\sin \omega t \cos \phi - \cos \omega t \sin \phi) \\
&= V_o I_o (\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi) \\
&= V_o I_o \{\tfrac{1}{2} (1 - \cos 2\omega t) \cos \phi - \tfrac{1}{2} \sin 2\omega t \sin \phi\} \\
&= \tfrac{1}{2} V_o I_o \{\cos \phi - \cos 2\omega t \cos \phi - \sin 2\omega t \sin \phi\}
\end{aligned}
$$

Inserting in this last equation a large number, N, of successive values for t, spread evenly over an entire cycle, would give N equations. Add all the left-hand sides and equate to the sum of the right-hand sides, and then divide both sides by N.

On the left is then the average value of the power taken over a complete cycle, and on the right is the average value

of $\frac{1}{2}V_o I_o \cos\phi$, the average value of $\cos 2\omega t \cos\phi$ and the average value of $\sin 2\omega t \sin\phi$.

The average value of the sine or cosine over a cycle is zero, so only the first quantity remains, hence,

$$\text{the average power} = \tfrac{1}{2}V_o I_o \cos\phi \text{ watts}$$
$$= \tfrac{1}{2}\sqrt{2}V\sqrt{2}I \cos\phi \text{ watts}$$
$$= VI \cos\phi \text{ watts}$$

Power-factor

The product of the R.M.S. values of volts and amperes is often called the apparent power, and the factor by which it must be multiplied to give the true power, $\cos\phi$, is called the power-factor.

$$\text{Power-factor} = \frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volts} \times \text{amps.}}$$

CHAPTER XIII

CONDENSERS AND CAPACITY

Condensers

THE condenser has considerable importance in A.C. work. Calculations relating thereto are more readily understood if there is some picture before the mind of what is taking place in the condenser.

Considering the present incomplete state of knowledge as to the exact nature of electricity, the picture need not necessarily be one which would be accepted by all, so long as it is of assistance to those who have no better one of their own. The following picture of what is taking place is made use of here.

Suppose that one has a battery of accumulators giving 200 volts, and that the mid-point of the battery is earthed; further, that one has two large flat insulated metal plates distant from one another and from surrounding objects.

Connect the plates, each to one terminal of the battery through insulated wires.

At the negative terminal of the battery the electrons are crowded together, the potential being – 100 volts. On joining the wire from the plate to this terminal, electrons project themselves off the terminal into the wire, and so on to what will now become the negative plate, until the potential of this plate becomes also – 100 volts.

On joining the wire from the other plate to the + terminal, electrons project themselves out of the end of the wire on to the + terminal, where there is a deficiency of electrons, the process continuing until wire and plate have parted with as many electrons as brings their potential to that of the + terminal, + 100 volts.

Evidently, during the above happenings, electrons have left the + plate and passed through the conductor with the E.M.F. in it, and collected themselves on the negative plate.

The quantity of electricity concerned in this transfer is small, since the plates cannot accommodate but a very small quantity of electricity for a potential of 100 volts.

Suppose now that the plates are brought very close to one

another, so that their flat surfaces are very nearly, but not quite, in contact.

It is now easier for the battery to discharge negative electrons on to the negative plate, since, although the negative charge there repels them as they approach, the positive charge attracts them, and it is now only the difference in the values of these two forces which resists their approach.

Similarly, it is also easier for negative electrons to be attracted away from the positive plate along the wire to the positive battery terminal, because the effect of the attraction of the + charge on the positive plate, which held them back before, is now largely neutralized by the force of repulsion from the negative charge on the other plate. This latter force tends to drive negative electrons off the positive plate, robbing some of the atoms of the plate of one of their negative electrons.

The result of bringing the plates near to one another is that negative electrons leave the positive plate, pass through the battery, and collect on the negative plate. Both plates now have increased charges, and consequently the forces which were brought into play, when one plate was brought near to the other, are now increased, and a quantity of negative electrons is again enabled to leave the positive plate and pass through the battery to the negative plate. This has the same effect over again, and a comparatively large quantity of electricity is transferred from the one plate to the other before equilibrium is again established. The operations considered above take place instantly if there is nothing in the circuit, from plate to plate, to impede the passage of the electrons.

The combination of the two or more plates, with their separating dielectric, is called a condenser.

Capacity of a Condenser

When a battery, or other source of E.M.F., is connected to the terminals of a number of different condensers in turn, the quantities of electricity transferred from the one plate to the other may have quite different values. It is said that the condensers have different capacities. A definition of capacity is evidently required.

The capacity of a condenser, in farads, is defined as being

equal to the quantity of electricity Q, in coulombs, which is transferred from one side of the condenser to the other, per volt E.M.F. of a battery connected directly across the terminals of the condenser.

The capacity in farads is usually denoted by the letter C. The farad is a very large unit, and the majority of condensers have capacities only amounting to a very small fraction of a farad.

Evidently, $$C = \frac{Q}{V} \text{ and } Q = CV$$

Condenser Connected to an Alternator

The value of the current i, at any instant, is equal to the rate of change of Q, the quantity of electricity which has been transferred from one side to the other of the condenser, for suppose at some instant Q to have the value 0·1 of a coulomb, and that a $\frac{1}{200}$ part of a second later the value is zero, then 0·1 coulombs must have passed through the connection from one side to the other of the condenser, in a $\frac{1}{200}$ part of a second. The average rate of flow is obviously 20 coulombs per second, i.e. the average value of the current is 20 amp. But since Q is equal to CV, the rate of change of the value of Q must be equal to the rate of change of CV; V is supposed to be a sine quantity, having a maximum value V_o. If a vector of length proportional to the value of CV_o, rotated at f r.p.s., the height or depth of its tip would give the value of Q to scale, and the rate of change of the height or depth would give the value of the current to scale.

The maximum value of the rate of change of a sine quantity was discussed in Chapter XII (see Fig. 38). The maximum rate of change of the height or depth of the tip of the vector CV_o is equal to $2\pi fCV_o$ volts per second, hence the maximum value of the current flowing through the alternator in and out of the condenser has the value $2\pi fCV_o$ amp. Then,

$$I_o = 2\pi fCV_o, \text{ or in R.M.S. values } I = 2\pi fCV$$

The equation can be put into a form similar to that used for finding the value of the current in a circuit containing resistance or reactance, and it becomes

$$I = \frac{V}{\frac{1}{2\pi fC}} \text{ amp.}$$

The quantity by which the number of the volts must be divided to get the value of the current, $\frac{1}{2\pi fC}$, is called the capacity reactance. Its value is seen to depend not only on that of the capacity, but also on that of the frequency.

The above result could have been obtained easily if slightly more advanced mathematics had been made use of, thus,

$$i = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{d(V_o \sin 2\pi ft)}{dt} = 2\pi fCV_o \cos 2\pi ft$$

Hence, $I_o = 2\pi fCV_o \qquad \therefore\ I = 2\pi fCV$

and $$I = \frac{V}{\frac{1}{(2\pi fC)}} \text{ amp.}$$

The current is seen to be a cosine function, and to be 90° out of phase with the alternator volts.

That the current would be 90° out of phase with the volts at the terminals of the condenser would be expected, since when the volts are just at their maximum value for a very short time there is scarcely any change in their value, and therefore the current is zero. The volts being at their maximum value, the condenser has its maximum charge. As the voltage falls, the charge leaves the condenser, the current being proportional to the rate of change of the volts. This latter rate of change is greatest as the volts are passing through their zero value, and the current has its greatest value, the condenser completing its discharge as the volts approach zero and instantly commencing to charge in the opposite direction as the volts pass through the zero point, the current concerned being all in the one direction. These changes can be followed with reference to Fig. 43.

In the vector diagram of Fig. 43, the current vector is shown 90° in front of the voltage vector. That this is correct can be understood from the following reasoning. Returning to the conventional idea of the flow of the positive electricity, the E.M.F. has full positive value at the instant depicted in the vector diagram, and must consequently have just finished driving the full charge of positive electricity on to the upper plate, shown in the diagram of connections. An instant later than that shown in the vector diagram, the alternator E.M.F.

will have a slightly lesser value, and out will come part of the positive charge collected on the upper plate, flowing back through the alternator in the direction opposed to that chosen as positive and marked on the diagram, meaning that, as the alternator E.M.F. passes through its maximum positive value, the current passes through its zero towards negative values.

The fact that the current leads has considerable importance in Electrical Engineering, and large condensers are made use of for power-factor improvement.

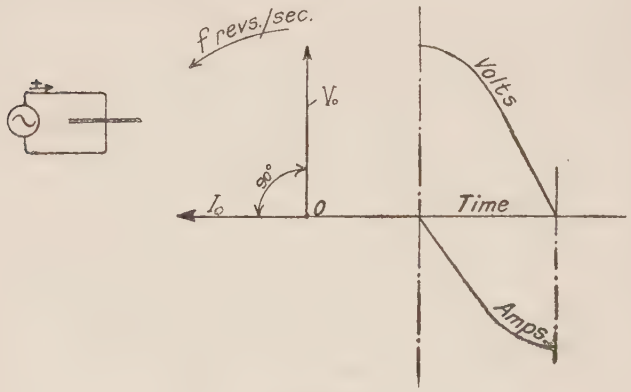

FIG. 43

The Condenser in Series with Resistance or Inductance, or Both

Kirchoff's Law is enunciated (see Chapter IV) for any closed electric circuit. A circuit, containing a condenser in series, is not a closed one. The law might still be applied for the purpose of finding the value of the current in such a circuit, if the condenser were thought of as being replaced by a conductor containing an E.M.F., of such value and direction at all instants, as would give to the condenser terminals precisely the same potential as that which they do actually have with the condenser present. The current would then be thought of as flowing through only for the purpose of determining its value. It is interesting to notice that this is not so very different from what does actually take place, since in

the ordinary operation of a condenser, the arrival of a batch of electrons on to the one plate is accompanied by the departure of a similar number of electrons from the other plate.

Under the above supposed conditions, Kirchoff's Law can be applied to find the value of the current, and will give the correct result.

The maximum value of the E.M.F., to be thought of as introduced in place of the condenser, will be equal to

$$I_o\left(\frac{1}{2\pi fC}\right)$$

Adhering to the conventional idea of the flow of the positive

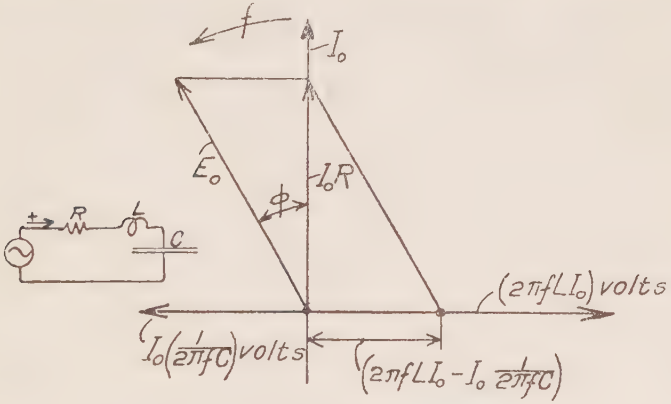

FIG. 44

electricity, the E.M.F. must be thought of as being directed from within towards that terminal which is positive at the instant concerned. This consideration fixes the position of its vector as being 90° in front of the current vector.

In drawing the vector diagram, commence by assuming the maximum value of the current I_o, and work back to find the value which the alternator E.M.F. must have to correspond.

Mark off I_o in any position to the scale chosen for amperes. On it, to the volt scale chosen, mark off the length I_oR.

Set off, 90° behind the current vector, the length $2\pi fLI_o$, to the volt scale, to represent the E.M.F. of self-inductance.

Then set off a length equal, on the volt scale, to $I_o\left(\frac{1}{2\pi fC}\right)$, 90° in front of the current vector, to represent the E.M.F., supposed to replace in effect the condenser. The alternator E.M.F. (vector) is now the only one which is missing in the diagram, and its value and position can be found, because the algebraical sum of all the three E.M.F.s must at all instants be equal to the value of the IR product, so the vector representing the algebraical sum of these E.M.F.s must coincide in position and length with the vector I_oR.

The lengths of the two E.M.F. vectors in line with one another can be directly subtracted, the one from the other. The remainder, combined with the vector of the unknown E.M.F., must give a vector coinciding with I_oR.

Thus the value and position of the vector E_o is fixed.

From the geometry of the diagram, the relationship of the vectors to one another can be seen, hence the following equation,

$$E_o^2 = (I_oR)^2 + \left(2\pi fLI_o - I_o\frac{1}{2\pi fC}\right)^2$$

$$= I_o^2\left[R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2\right]$$

Transposing and taking the square root of both sides,

$$I_o = \frac{E_o}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

Multiply by 0·707 to get R.M.S. values, then

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \text{ amp.}$$

also $$\tan\phi = \frac{\left(2\pi fL - \frac{1}{2\pi fC}\right)}{R}$$

The expression by which the volts are divided to obtain the value of the current is, as before, called the impedance: it now contains an additional term on account of the capacity in series. If the capacity term, on inserting the values concerned, turns out to be larger than the inductance term, the minus sign thus introduced disappears on squaring.

If the two terms happen to be equal, the impedance is thus reduced to that of the resistance term only, and what is called resonance occurs in the circuit. If the resistance is very small, a very large current for but few alternator volts is obtained. This is referred to later under condition for resonance.

It will be noticed that if the vector of the condenser volts is longer than that of the E.M.F. of self-inductance, the current will lead with respect to the alternator volts.

EXAMPLE. Find the value of the current which will flow in a circuit containing an alternator giving an E.M.F. of 500 volts, a resistance of 10 ohms, an inductance of 0·15 of a henry, and a condenser of 100 mfd. capacity, all in series. The frequency is 50 cycles a second.

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{500}{\sqrt{100 + \left(314 \times \frac{15}{100} - \frac{1000000}{314 \times 100}\right)^2}}$$

$$= \frac{500}{\sqrt{100 + 234}} = \frac{500}{18{\cdot}3} = 27{\cdot}3 \text{ amp.}$$

If a circuit has either no capacity, or no resistance, or no inductance, the corresponding term in the expression for the impedance becomes zero, and the expression still gives the true value of the impedance.

Condition for Resonance

As stated above, resonance will occur in a circuit having capacity and inductance in series, the inductance and capacity

terms neutralizing one another in the expression for the impedance, thus

$$2\pi fL = \frac{1}{2\pi fC}$$

$$\therefore\ 4\pi^2 f^2 LC = 1$$

$$\text{and}\quad f^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore\ f = \frac{1}{2\pi\sqrt{LC}}$$

In wireless reception, the value of C is often very finely adjusted, so that resonance occurs exactly at the frequency of the desired sending station.

Although the E.M.F. of the alternator may be small, if the resistance is low enough, quite large voltages will be found across the terminals of the condenser and the inductance, and considerable amounts of energy are being transferred from the condenser to the inductance and back to the condenser again. One might compare it with the swinging of a pendulum, under the action of small impulses, which happen to be in exact agreement with the natural time of swing of the pendulum. Each small impulse helps to build up a store of energy possessed by the pendulum. As the amplitude of the swing becomes greater, the dissipation of energy per swing generally becomes greater, until it is large enough to just dissipate the energy being received. In the electrical case, the R.M.S. value of the current goes on increasing in value, until the losses in the circuit are large enough to just dissipate the energy being received from the alternator. If the resistance is low enough, the above value of the current may be quite large.

Two or More Impedances in Series

EXAMPLE. Find the value of the current which will flow through a circuit made up of an alternator giving 500 volts at 50 cycles, connected in series with two impedances, one of 5 ohms and 30° lag at 50 cycles, and the other of 8 ohms and 60° lag at 50 cycles.

The solution can be found very easily by a graphical method. The current passing through the two impedances is the same.

Assume any value of current, and find the corresponding value of the alternator volts. The value of the current being then known for one voltage, it can, by simple proportion, be found for any other.

Assume, say, 7·07 amp. The maximum value will then be 10 amp. The R.M.S. volts at the terminals of either impedance will be equal to the value of the current multiplied by the value of the impedance concerned, for

$$Z = \frac{\text{volts}}{\text{amps.}} \qquad \therefore \text{ volts} = \text{amps.} \times Z$$

This obviously holds for maximum values too. The maximum value of the volts on the terminals of the 5 ohm impedance

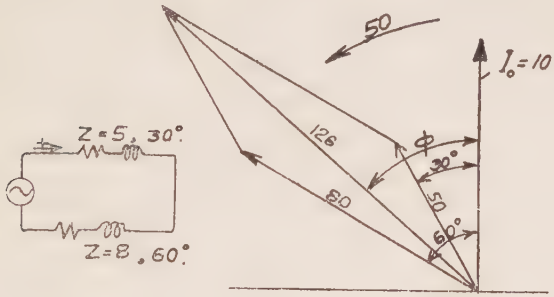

FIG. 45

is 50, and as the current lags 30° behind the volts, the volt vector will have the position shown in the diagram. Similarly, the value and position of the 80-volt vector is found. As these two vectors rotate, the height and depth of their tips at any instant give the P.D.s between the terminals of the two impedances.

Complete the parallelogram. The height or depth of the tip of the diagonal is equal to the algebraical sum of the heights and depths of the tips of the 50- and 80-volt vectors. But the alternator volts are at every instant equal to the algebraical sum of the two instantaneous values of P.D.s between the terminals of the impedances, so the diagonal represents the maximum value of the alternator volts correctly.

The maximum value of the alternator volts is found to be 126, so the R.M.S. value will be 0·707 × 126 or 89·2 volts, then

$$89{\cdot}2 \text{ volts send } 7{\cdot}07 \text{ amp.}$$

$$\therefore \quad 500 \text{ volts send } 7{\cdot}07 \times \frac{500}{89{\cdot}2} = 39{\cdot}6 \text{ amp.}$$

The total value of the impedance of the circuit is equal to the value of the ratio volts to amperes, so has the value $\frac{500}{39{\cdot}6} = 12{\cdot}6$ ohms.

The angle of lag ϕ can be measured off the diagram. If the two values 5 and 8 ohms had been added directly, the sum would have been 13.

Impedances cannot be added directly unless they have the same angle of lag or lead, or the same phase angle as it is called. The more different the phase angles, the greater the error in adding directly.

An impedance diagram may be drawn which is not a vector diagram, i.e. does not rotate.

Thus for the 8- and 5-ohm impedances, if a line of length 8 and another of length 5 had been drawn at 30° inclination to one another, from any point, on completing a parallelogram with these as adjacent sides, the diagonal would have given the correct sum of the two impedances.

This summation can be extended to any number of impedances.

Evidently in specifying the value of an impedance, the frequency for which it has this value should be stated, also particulars as to the angle of lag, or the power-factor of the impedance are necessary, before it is completely defined.

Suppose the power-factor were given as 0·5. This means that the product of the R.M.S. volts supplied to the impedance and the R.M.S. amperes has further to be multiplied by 0·5 to give the power supplied to the impedance. For sine currents and voltages the power-factor = cos ϕ, so cos ϕ = 0·5, and ϕ has therefore the value 60°.

If instead of giving the value of each impedance the value of the resistance and inductance parts of each are given, the values of the reactances can be found ($2\pi fL$) and added together to form a single reactance, also the resistance parts

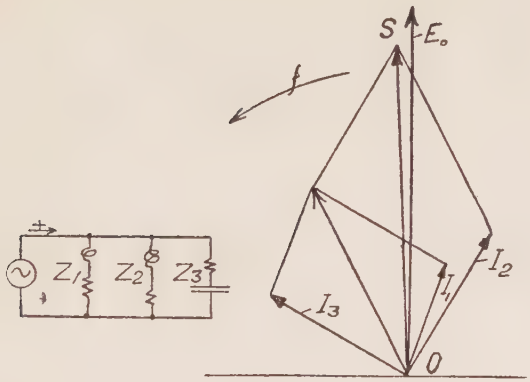
S
E_o
f
Z_1
Z_2
Z_3
I_3
I_1
I_2
0

FIG. 46

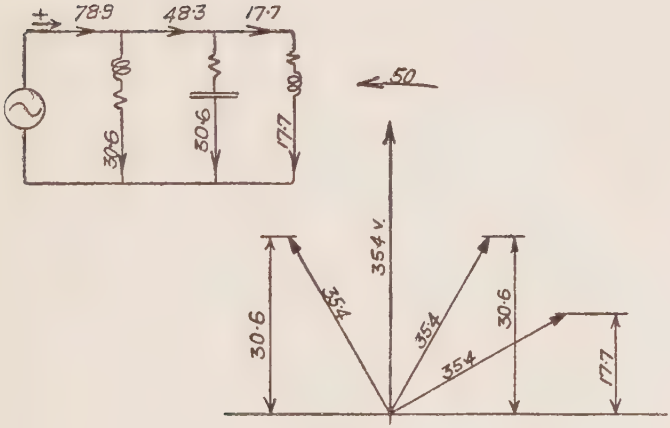
78·9
48·3
17·7
30·6
50·6
17·7
50
354 V.
45·4
354
354
354
30·6
30·6
17·7

FIG. 47

can be added to form a single resistance, and the value of the total impedance found, thus

$$Z = \sqrt{(R_1 + R_2 + R_3, \text{ etc.})^2 + (X_1 + X_2 + X_3 + \text{etc.})^2}$$

The above depends on the fact that in such a circuit the position of the resistances and reactances in the circuit has no influence on the value of the current for a given alternator E.M.F.

Impedances in Parallel

Let Z_1, Z_2, and Z_3 be three known impedances in parallel They all receive the same P.D. at all instants. The values of the current for each can be obtained, for

$$I_1 = \frac{E}{Z_1}, \; I_2 = \frac{E}{Z_2}, \; I_3 = \frac{E}{Z_3}$$

The angular position of the three current vectors will be known from the particulars of each impedance. Add the currents vectorially: the sum, represented by *OS* in Fig. 46, is equal to the value of the alternator current.

The equivalent impedance of the group of parallel impedances can, of course, be found, if desired, for it has the value equal to alternator volts divided by alternator amperes, and the value of the amperes has just been found.

EXAMPLE. Three impedances, each of value 10 ohms at 50 cycles, are connected in parallel to the terminals of an alternator giving 250 volts at 50 cycles.

The power-factors are 0·5 and 0·866 lagging and 0·866 leading.

Draw a diagram of connections and mark in on it the values and directions of the currents everywhere, at an instant when the alternator E.M.F. is passing through its maximum value.

All the information can be obtained from a vector diagram drawn in the position of maximum alternator E.M.F. (See Fig. 47.)

CHAPTER XIV

POLYPHASE WORKING

Three-phase Circuits

THERE are three similar windings in a three-phase alternator, in each of which an E.M.F. of the same R.M.S. value is generated. The E.M.F.s are out of time with one another by 60° or 120°, according to how the windings are connected. If the windings are connected in star, as in Fig. 48, and the

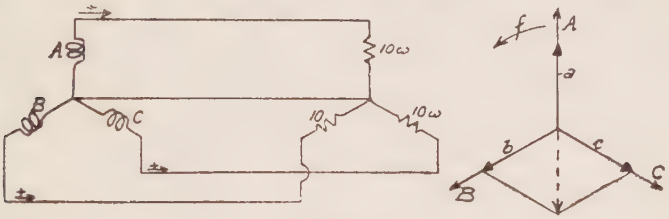

FIG. 48

correct ends are taken to the star, the three E.M.F.s will be 120° out of phase with one another with respect to a direction chosen as positive, outwards from the star point to the line, for all three.

Suppose in Fig. 48 the E.M.F. in each winding, or the phase E.M.F. as it is called, has the R.M.S. value 200 volts.

Applying Kirchoff's Law to the circuit consisting of the winding of phase *A*, the corresponding load arm and the middle wire, the current through the resistance is seen to have the value 20 amp. and to be in time, or in phase, with the E.M.F. Similarly, considering the other two circuits through windings *B*, and then *C*, the currents through all the load arms have the value 20 amp., in each case in phase with the corresponding E.M.F. The line currents are 20 amp. To find the value of the current in the middle wire, the algebraical sum of the currents meeting at any instant at the star point of the load must be found. Calling them in each case

positive when flowing towards the star point of the load, the three-current vectors are *a*, *b*, and *c* in Fig. 48.

The diagonal of the parallelogram, of which *b* and *c* are adjacent sides, is equal in length to *a*, and the algebraical sum of the three currents is, at all instants, zero.

Since no current flows in the middle wire, it can be removed without in any way affecting anything in the circuit, so long as the three E.M.F.s have equal R.M.S. values, are 120° apart, and the load is balanced, i.e. all three load arms have exactly equal impedance.

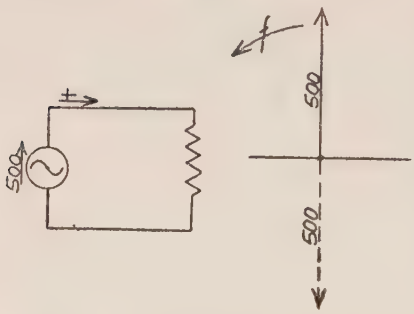

Fig. 49

When any calculations are being made with balanced loads, it simplifies matters if the middle wire is considered as being present. The equations are then very simple ones.

The power-factor may have any value for an arm of the load, according to the nature of the load, but it must be alike for all three arms of any balanced load. Unbalanced loads are dealt with later.

Reversal of Vector

A vector diagram should always be accompanied by a corresponding diagram of connections, and on that diagram the direction chosen as positive should always be marked. Also the direction of rotation of the vectors should be indicated, so that the whole is intelligible, at a glance, to anyone.

If it is desired to reverse the direction chosen as positive, the corresponding vector or vectors must be reversed, and all is quite correct again.

Suppose, for example, that the alternator in Fig. 49 is giving, at the instant considered, its maximum value of 500 volts upwards on the diagram of connections.

The vector diagram represents this correctly, for, according to it, the volts have at the moment their positive maximum of 500.

Reverse the sign chosen as positive on the circuit diagram. The vector diagram still informs one that the volts are at their positive maximum value of 500, i.e. that the E.M.F. has the direction downwards through the alternator. This is quite wrong, for it actually acts in the upward direction.

Now reverse the E.M.F. vector as shown dotted. The vector now tells the truth.

Hence it is learned that a reversal of the sign chosen as positive on the circuit must be accompanied by a reversal of the corresponding vector or vectors.

Immediate use will be made of this in the following paragraph.

Line Voltage in Terms of Phase Voltage

In Fig. 50 the three vectors, A, B, and C, represent the three E.M.F.s in the three windings, taken as positive in the

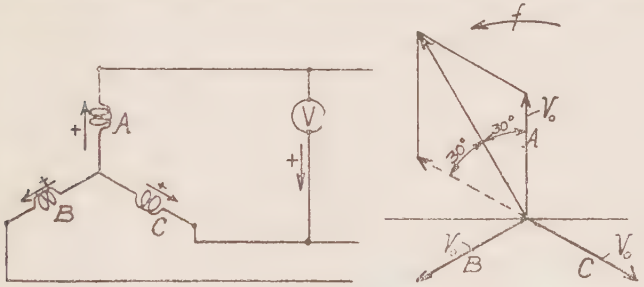

FIG. 50

directions from star point to line. If the generator has been connected up correctly in the winding shop, these three E.M.F.s are 120° a part, as shown. Calling the maximum value of the phase volts V_o, find the maximum value of the line volts. Find the algebraical sum of the E.M.F.s for the circuit through windings A and C, and through the voltmeter. Choose positive direction for this circuit as indicated beside the voltmeter.

The vector A, of length V_o, will do, but in the light of the previous paragraph, the direction chosen as positive for the E.M.F., C, could be reversed if the vector C were reversed at the same time.

The E.M.F.s can then be added vectorially.

The vectorial sum is $\sqrt{3}$ times V_o.

If this is true for maximum values, it will also be true for R.M.S. values.

The same method applies to finding the voltage between any of the three pairs of lines A and B, B and C, and C and A.

With the star connection, then, the line voltage is $\sqrt{3}$ times the phase voltage, and the line current is equal to the phase current.

Three-phase Power (Star)

With balanced loads, each arm of the load gets equal power, and the total power is equal to three times the power for one arm.

If the volts supplied to each arm of the load are out of phase by an angle ϕ with the current flowing through that arm of the load, the power is as follows—

Power per phase = ph. volts $\times$ ph. amps. $\times \cos \phi$

$$\begin{aligned} \text{Total power} &= 3 \times \text{ph. volts} \times \text{ph. amps.} \times \cos \phi \\ &= 3 \times \frac{\text{line volts}}{\sqrt{3}} \times \text{line amps.} \times \cos \phi \\ &= \sqrt{3} \times \text{line volts} \times \text{line amps.} \times \cos \phi \end{aligned}$$

EXAMPLE. Find the power given out by a star-connected three-phase generator, giving 300 line volts at 50 cycles, when connected to a load, consisting of three impedances of value 17·3 ohms each, at 50 cycles, the power-factor being 0·866 for each. The impedances are star-connected.

The diagram of connections would be similar to that in Fig. 48.

The line volts being 300, the phase volts are 173, and thus the current per phase has the value 10 amp.

$$\begin{aligned} \text{The total power} &= \sqrt{3} \times \text{line volts} \times \text{line amps.} \times \cos \phi \\ &= \sqrt{3} \times 300 \times 10 \times 0{\cdot}866 \\ &= 4500 \text{ watts.} \end{aligned}$$

Mesh or Delta Connections

The three windings of the alternator are connected as in Fig. 51.

In connecting up, the ends to be joined must be so chosen that the three generated E.M.F.s are 120° apart, with respect to directions chosen as positive, the same way round the triangle.

The resultant E.M.F. round the mesh will, with sine wave-shapes, be, at all instants, zero.

Kirchoff's Law may be applied to a circuit through one of the E.M.F.s and through the corresponding load arm, or alternatively through that load arm and back through the other two E.M.F.s. In both cases, the R.M.S. value of the

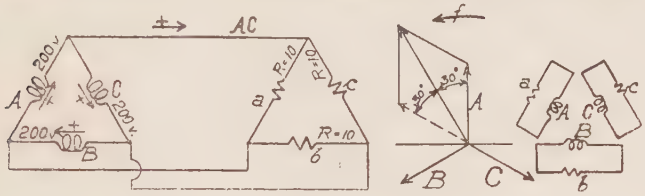

FIG. 51

current would be found to be 20 amp. through that resistance, and the same value would be found for each resistance.

The little diagram on the right in Fig. 51 helps to make one realize that phase *A* is really directly connected to load arm *a*, and so on for the others, thus the current through load arm *a* depends only on the volts of *A*.

In the complete diagram on the left, the current in any load arm can be considered as coming exclusively from its own winding on the generator. Actually the currents from two phases mix in the line; but since the values of the current in one phase winding and corresponding load arm are at all instants equal, it is of no moment whether the same individual electrons pass through load arm *a*, which have passed through winding *A*.

Three-phase Power (Mesh Connections)

Add vectorially the currents from phase *A* and phase *C* passing through line *AC*, the direction chosen as positive being marked in the diagram. Since the load is non-inductive, the current through phase *A* will be in phase with the volts. The volt vector in the diagram can also be used, to an ampere

scale, to represent this current. Similarly, the volt vector C can be made use of; but since the sign belonging to it does not agree, reverse the sign and at the same time the vector. This reversed vector now represents correctly the currents flowing in phase C, the positive direction being through the winding towards the line AC.

The addition of the two currents at all instants now gives the instantaneous value of the current in the line. The R.M.S. value of the current in the line is $\sqrt{3}$ times the R.M.S. value of the phase current, and as before,

$$\begin{aligned}\text{Total power} &= 3 \times \text{power in one load arm} \\ &= 3 \times \text{phase volts} \times \text{phase amps.} \times \cos\phi \\ &= 3 \times \text{line volts} \times \frac{\text{line current}}{\sqrt{3}} \times \cos\phi \\ &= \sqrt{3} \times \text{line volts} \times \text{line amps.} \times \cos\phi\end{aligned}$$

With the numbers in Fig. 51, the phase current is 20 amp., and therefore the line current is 34·7 amp.

$$\begin{aligned}\text{Power} &= \sqrt{3} \times 200 \times 34{\cdot}7 \times 1 \\ &= 12000 \text{ watts.}\end{aligned}$$

That the power expression is the same for a star or a mesh load is convenient, when, as is often the case with three-phase induction motors, it is not known how the three windings have been connected.

EXAMPLE. Find the line current taken on full load by a 100 b.h.p., 500-volt, 3-phase induction motor, if the efficiency for that loading is 0·93 and the power-factor 0·88.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{100 \times 746}{\sqrt{3} \times 500 \times I \times 0{\cdot}88} = 0{\cdot}93$$

$$\therefore \text{ the line current } I = \frac{100 \times 746}{\sqrt{3} \times 500 \times 0{\cdot}88 \times 0{\cdot}93} = 105 \text{ amp.}$$

Watt and Wattless Components of a Current

It is sometimes useful to think of the actual current as being separated into two parts, or components, one in phase with the volts and the other 90° out of phase with them.

Suppose an alternator to be connected to a synchronous motor, and the current to lag by an angle ϕ behind the alternator volts.

If it were desired, the actual current I, represented by the vector OC, could be thought of as divided into two currents represented by the vectors OA and OB. At any instant, the algebraical sum of the two component currents would be exactly equal to the actual current. The component in phase

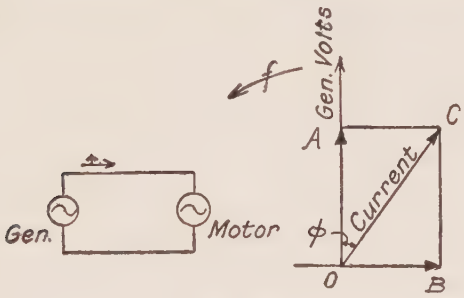

FIG. 52

with the volts would evidently have the value $I \cos \phi$, and that out of phase $I \sin \phi$.

The watt component is often called the power component of the current, since if it is multiplied by the volts, the result is $V \times I \times \cos \phi$, which is identical with the expression for the power.

An example of the use of the above is given below.

EXAMPLE. Suppose a group of 1,000-volt, 3-phase induction motors take a current of 100 amp. through a rather long three-core cable, the power-factor being 0·707 lagging. Find what current would have to be taken by each of three condensers, connected between the lines near the load, so that the power-factor of the whole is improved from 0·707 to 0·9063.

Suppose the connections are as shown in Fig. 53. OI represents the current taken by the load per phase, and OB its watt and OA its wattless component. After power-factor correction, the watt-component must remain as before, because the power supplied is unaltered in value.

The wattless component OA has the value 100 sin 45°, or

70·7 amp., and with 45°, the watt-component also has the value 70·7 amp.

When the power-factor has been improved to 0·9063, corresponding to 25° lag, the value of the wattless component, *OD*, will be equal to 70·7 tan 25°, or 70·7 × 0·4663 = 33 amp. The change in the wattless component from 70·7 to 33 amp. is 37·7 amp., and if the condensers each take a current of 37·7 amp., this current *OC*, combined with the load current *OI*, would give the current *OE* in the cable; and as far as the

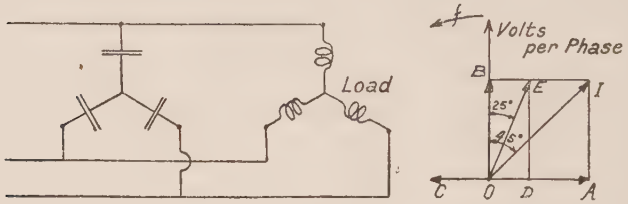

Fig. 53

cable and supply are concerned, the power-factor of the load is now 0·9063 instead of 0·707.

R.M.S. Values in Vector Diagram

It saves trouble to use these values, so long as it is remembered that, if the vectors have only 0·707 of their correct length, the heights or depths must be multiplied by $\sqrt{2}$ to give, on the R.M.S. volt scale, the correct instantaneous values. For example, suppose a current of 100 R.M.S. amp. is represented by a vector 100 mm. in length, the scale being 1 mm. to the R.M.S. amp., then, if this vector is passing through a position 45° from the vertical, the height would be 70·7 mm. and the instantaneous value of the current $\sqrt{2} \times 70{\cdot}7 = 100$ amp.

Quite often, where a number of vectors are to be combined, no instantaneous values are actually wanted, and it saves much time to use R.M.S. values for the lengths of the vectors.

Distributed Capacity

A long high-voltage cable has capacity, and the charging current may be considerable.

Suppose, for instance, in a 10,000-volt single-phase case, the charging current had the value 15 amp., then, if the load supplied through the cable took a current of 50 amp. at a power-factor 0·809, the current supplied by the alternator would be equal to the vectorial sum of 15 amp. leading 90°, and 50 amp. lagging 36°. This will be found to be 43 amp., and the power-factor at the alternator is 0·942 lagging.

Current Circulating Between Two Three-phase Alternators not Quite in Step

Suppose both machines unloaded. Consider the circuit marked *X*. *OA* represents the E.M.F. generated in the phase of machine *A*, and *OB* that of machine *B*. For exact synchronism these two would be equal and opposite, and no current would circulate between the two machines. (See Fig. 54.)

Suppose the rotor of machine *B* to have dropped behind by a small angle θ, shown in the vector diagram. There is now a small resultant E.M.F. in the circuit *X*, indicated by *OR*, and according to Kirchoff's Law the current in that circuit will be dependent only on the value of the resultant E.M.F.

The circuit through the two windings will contain perhaps about four times as much reactance as resistance and, if the exact figures are not given, it can be assumed that the current will lag by about 75° or so behind the volt vector, and hence the current vector is not far from being in phase with the E.M.F. of machine *A*. If the reactance and resistance of the windings are given and the voltage of the alternators, the value of the resultant volts can be at once found from the vector diagram; and dividing this by the impedance of circuit *X*, the value of the circulating current, corresponding to the angle θ, is determined. In this, no account is taken of the armature reaction in the alternators. If, instead of the ordinary impedance of the stator windings, the so-called synchronous impedance had been taken, this would have taken armature reaction, to a certain extent, into account, and the value obtained for the circulating current would have been more nearly correct.

Example. Particulars for above. Both machines alike: 6,600 line volts 50 cycles. Phase displacement 5°. Synchronous impedance of each such as to take up 10 per cent of the

phase volts, when full-load current circulates. Find the circulating current in terms of the full current.

The resultant E.M.F. has a value very nearly equal to $OA \sin 5° = 0{\cdot}0872\ OA$. Two windings are in series, so $0{\cdot}2\ OA$ would be required to send full current, therefore the fraction of full current circulating $= \dfrac{0{\cdot}0872}{0{\cdot}2} = 0{\cdot}436$.

It should be carefully noted that the transference of energy between the machines has nothing directly to do with the

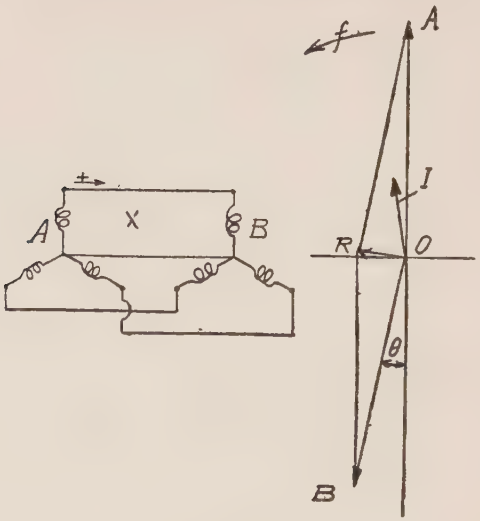

Fig. 54

value of the resultant E.M.F. This is no real E.M.F. itself; it is only the value of the difference between two large E.M.F.s.

A coulomb, in passing through the windings of one phase, for unity power-factor receives or gives up 3,810 joules. The phase volts are 3,810. Thus for a number of cycles, before the angle θ has had time to change appreciably, machine A gives out nearly 43·6 per cent of its full output to machine B; thus powerful torques are exerted on the rotors of the machines, tending to retard the one and accelerate the other, until there is a state of exact synchronism.

Unbalanced Three-phase Loads

There is no difficulty in calculating the values of line currents if the load is connected in mesh.

Suppose two of the phase currents are as in Fig. 55, and that it is required to find the line current. The vectors *OAB* and *OCA* represent the voltages for the arms of the load *AB* and *CA*, reckoned positive in the directions shown inside the mesh. The current vectors can now be inserted. Take the positive direction for line currents as being towards the load.

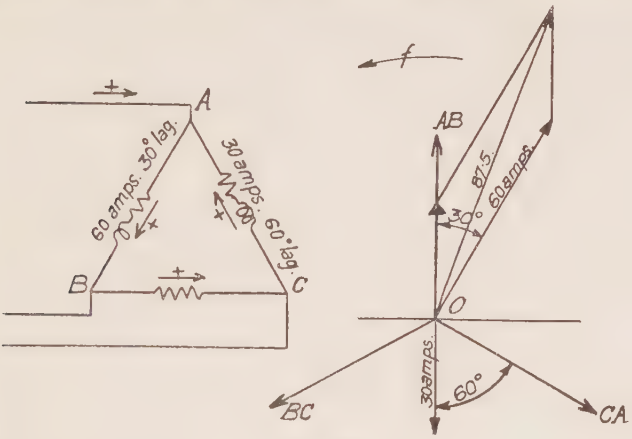

Fig. 55

The vector for the 30-amp. current is reversed to correspond with this sign, and the vectorial addition shows the line current to have the value 87·5 amp.

Similarly, the values of the other two-line currents can be found.

The ordinary expression cannot be used for finding the value of the power given to the three-phase load. The value for each phase must be found, and the three results added, to give the total power.

Unbalanced Loads (Star-connected)

A method of dealing with the above is given in Part I, Chapter VI.

(*a*) Imagine the star points temporarily joined, and find the current in this middle wire.

(*b*) Next imagine the three E.M.F.s each reduced to zero and an alternating E.M.F. of the same frequency, and of any convenient value, introduced into the middle wire. Find the current through the middle wire.

(*c*) By simple proportion, find the value of the E.M.F., which will cause the middle-wire current in cases (*b*) to be exactly equal and opposite to that in case (*a*).

(*d*) When the above alternating E.M.F. acts in the middle wire in conjunction with the three original E.M.F.s in the

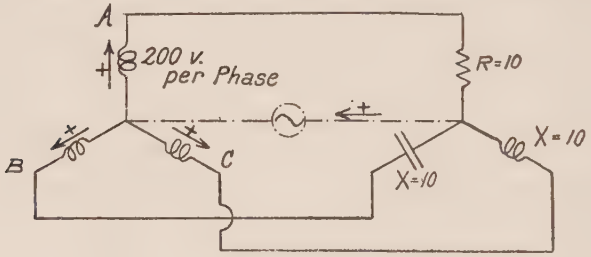

FIG. 56

star, no current will flow along the middle wire, but the finding of the values of the currents in the load arms is simplified and the voltage between the star points is equal to the value of the E.M.F. introduced.

The three alternator E.M.F.s are represented by *OA*, *OB*, and *OC*. Their currents, each of value 20 amp., are represented by *Oa* in phase with its volts, *Ob* 90° leading and *Oc* 90° lagging. The vectorial sum of these is represented by *OR* and has the value (34·63 – 20) or 14·63 amp., being positive when current is being delivered on to the star point of the load; the positive direction along the middle wire has been chosen to agree. At the moment depicted in Fig. 57 (*a*), the mid-wire current has its maximum value from left to right.

(*b*) Suppose mid-wire E.M.F. introduced has value 200 volts. The resultant current will be 20 amp. in phase with the E.M.F., since the condenser and inductance currents cancel each other.

(*c*) Only 14·63 amp. are required, so voltage must be reduced from 200 to 146·3 volts. Represented by *OM*.

(*d*) The vectors *OM* and *OA* add directly, and give 346·3 volts, resulting in 34·63 amp. through the resistance.

The condenser gets the vector sum of 146·3 and 200, 120° out of phase with each other, and the current is represented by *Obb*, leading by 90°.

Similarly, the current through the reactance coil is represented by *Occ*, lagging 90° behind the resultant E.M.F., *OCC*.

The vector sum of the currents *Obb* and *Occ* is equal to 34·63 amp., and thus no current flows along the middle wire.

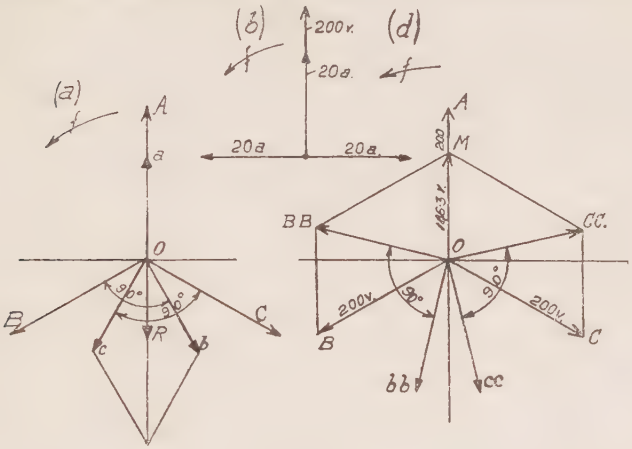

FIG. 57

The current in the resistance is 34·63 amp., and in both the reactance coil and the condenser it has the value of approximately 18 amp.

The voltage between the star points is 146·3. As no current flows through the middle wire, it can be removed without affecting the values of the currents.

The above is an example of how to deal with problems of unbalanced star loads.

If the reader has understood Kirchoff's Laws and what has been said about three-phase circuits here, he should be able to deal with any problem concerning them which might arise.

CHAPTER XV

NON-SINUSOIDAL CURRENTS

Transient Currents

KIRCHOFF'S Laws may be applied to find the values of these. Consider the circuit shown in Fig. 58. When the currents are quite steady in value, as shown, the switch S is suddenly opened. Find the values of the current in the inductance, from instant to instant, as the current dies away.

As the current dies away in the inductance, an E.M.F.

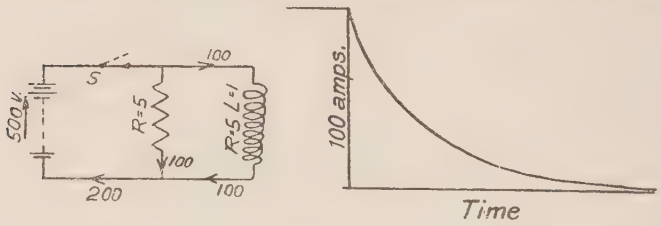

FIG. 58

equal to L times the rate of change of the current is produced in the direction of the current. This is the only E.M.F. in the closed circuit; then,

$$L \cdot \frac{di}{dt} = RI$$

$$\text{or } 1 \times \frac{di}{dt} = 10I$$

that is, the current dies away at a rate, in amperes per second, equal to 10 times the value of the current in amperes. When it has fallen to 50 amp., it is decreasing at the rate of 500 amp. per second, and so on.

Initial Values of Currents

As an example, consider the conditions in a circuit containing a resistance of 10 ohms, an inductance of $\frac{1}{30}$ henries, and

a sine E.M.F. of 50 cycles frequency and 1,000 volts maximum value, the circuit being closed at the instant of zero E.M.F.

So long as the current is increasing,

$$E - L\frac{di}{dt} = RI$$

The slope of the current curve, $\frac{di}{dt}$, will not alter very much during short intervals, and as a close approximation its value in the middle of a short interval might be taken as its average value during that interval. Also the value of the alternator E.M.F. at the middle of the interval might be taken as equal to its average value E_{av} during the interval. If δi is taken as the increase in the current during the interval, the equation for the middle of the interval, of length δt sec., is,

$$E_{av} - L\frac{\delta i}{\delta t} = RI_{av} = R\left(I + \frac{\delta i}{2}\right)$$

$$E_{av} - \tfrac{1}{30} \times \frac{\delta i}{\frac{1}{24} \times \frac{1}{50}} = 10I + 5\delta i$$

$$\therefore\ 45\delta i = E_{av} - 10I$$

$$\therefore\ \delta i = \frac{E_{av} - 10I}{45}$$

where I is the value of the current at the beginning of each interval, 24 intervals to the cycle being taken.

Interval (1). $\delta i = \dfrac{130 - 10 \times 0}{45} = 2{\cdot}9$ amp. (nearly)

Interval (2). $\delta i = \dfrac{380 - 10 \times 2{\cdot}9}{45} = 7{\cdot}8$ amp.

Interval (3). $\delta i = \dfrac{603{\cdot}5 - 10 \times 10{\cdot}7}{45} = 11$ amp.

The increases found must be added to the value, which the current had at the beginning of the interval.

During the ninth interval the increase is scarcely more than 1 amp., and during the tenth the alternator volts are less than those which would be required for the resistance, if the current kept the value, which it had at the beginning of this interval. Naturally the current commences to decrease, and this calls into play a self-induced E.M.F. assisting the current, and the sign in the equation must be changed. Then δi is a decrease, and

$$\delta i = \frac{10I - E_{av}}{45}$$

Later the alternator E.M.F. reverses, and for a short time the E.M.F. of self-inductance drives the current through the circuit against the alternator E.M.F. When the current reaches zero and begins to increase in the opposite direction, the E.M.F. of self-inductance still opposes the alternator E.M.F., but this latter has grown to values greater than it.

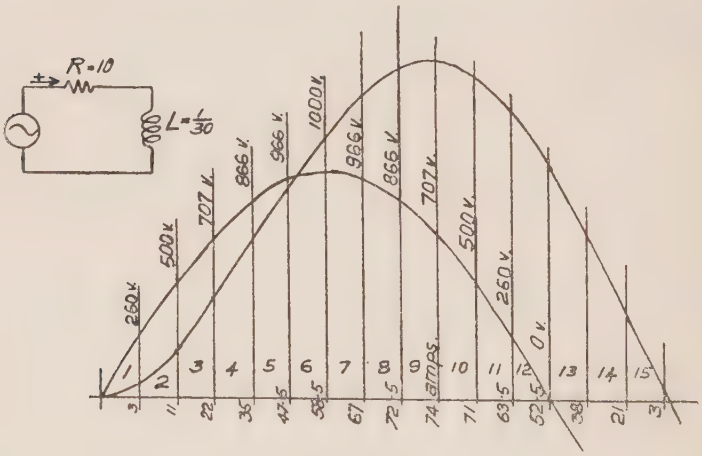

FIG. 59

The current curve will be found to be extraordinarily near to the correct place on crossing the zero line for the first time, i.e. after the first half-current wave, the others will be almost normal in shape.

Non-Sinusoidal E.M.F.s and Currents

Resistance Only. If the circuit contains nothing other than resistance, the current wave-shape will be the same as that of the E.M.F. It will be shown later how to find the R.M.S. values.

Inductance Only. Suppose the actual wave-shape of the current to be that marked *W*, the ordinates of this curve being in each case equal to the sum of the ordinates of the

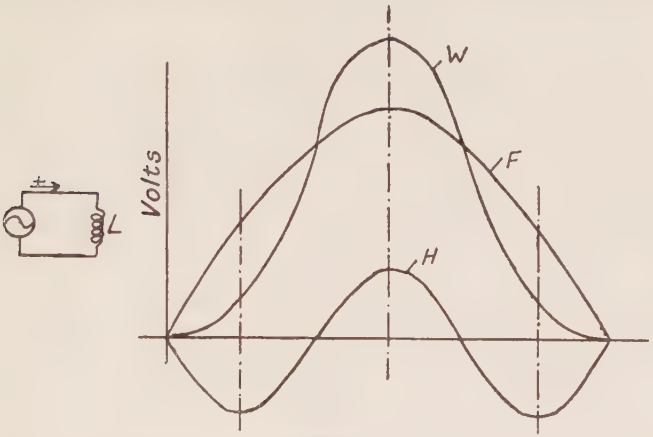

Fig. 60

two sine curves shown, of which *F* has the fundamental frequency and *H* exactly three times this frequency.

The rate of change of the value of the ordinates of the curve *W* is equal, at each instant, to the algebraical sum of the rates of change of the corresponding ordinates of curves *F* and *H*. This can be illustrated by taking numbers for the values of the ordinates at the beginning and end of a very short interval.

Let these be + 60 amp. for *W*, made up of + 50 and + 10 from *F* and *H*. At the end, suppose that the ordinate of *W* has become + 70, consisting of + 65 from *F* and + 5 from *H*. The rate of change for *W* is proportional to + 10, and that for *F* and *H* to + 15 and − 5. The algebraical sum of + 15 and − 5 is + 10.

Hence, at any instant, since

$$E = L\frac{di}{dt}$$

$$\therefore\ E = L\frac{dF}{dt} + L\frac{dH}{dt}$$

But $L\frac{dF}{dt}$ and $L\frac{dH}{dt}$ are each the instantaneous values, which an E.M.F. would have, if it were to send a current F or H through the inductance L.

Evidently the E.M.F., which would send the current, of wave-shape W, through the inductance L, would have a wave made up of the sum of two sine E.M.F. waves, one of fundamental and the other of triple frequency, the amplitudes of each being such as would send the currents F and H through the inductance separately, and their phase position also such as would give the current waves, F and H, their correct relative positions.

The reactance, for the triple frequency, will be three times as great as for the fundamental frequency.

Resistance Inductance and Capacity

If, in addition to the inductance, resistance is included in the circuit, it can be shown, in a similar manner, that the alternator E.M.F. wave is made up of the two sinusoidal E.M.F.s, one of which is able to send the current F, and the other the current H, separately through the impedance of the circuit.

The angles of lead of these E.M.F.s before their currents F and H will be different, since the resistance term is the same, but the reactance term is three times as great for the triple frequency.

Had capacity in series also been included, this would have altered the value of the impedance and the angles of lead of the two E.M.F.s, but the method could be used as before.

The equation for the alternator E.M.F. is then evidently of the form

$$E = A\ \sin \omega t + B\ \sin (3\omega t - \theta)$$

where A and B are the maximum values.

More Complicated Wave-shapes

Suppose the current wave to be made up of a sine curve of fundamental frequency, and of a number of odd multiple frequency (3, 5, 7, and so on) sine curves.

The same reasoning could be applied, and the E.M.F. wave would be found to be made up of the sum of a number of sinusoidal waves of fundamental and higher frequencies, each having such phase position and amplitude as would send its own current separately through the impedance of the circuit.

The higher frequency sine curves in the irregular current wave can have any amplitude and phase position relative to the fundamental sine curve, and there can be any number of them, each having a frequency which is an odd multiple of the fundamental frequency, and thus almost any shape of current wave could be built up and the E.M.F. required to give this current found.

The multiple frequency quantities are called harmonics. Only odd harmonics, 3, 5, 7, etc., occur in alternator E.M.F.s, because the even harmonics will be found to give the two half waves different shapes. In an alternator the one-half wave is generated under exactly the same conditions as the other, and however distorted the wave-form, both half waves are alike.

Effect of Capacity and Inductance on the Wave-shape

The higher harmonic components of an E.M.F. do not produce much effect in a circuit having considerable inductance, but on the other hand they can produce considerable currents where capacity is present. Resonance may occur for one of the harmonics, in which case the total impedance, as far as it was concerned, would have a value only equal to the resistance.

Since capacity causes leading harmonic current components and inductance lagging ones, and since inductance discourages and capacity encourages the value of harmonic currents, the wave-shape of the current, relative to that of the alternator E.M.F., is considerably different, for a circuit containing inductance, from that for a circuit containing capacity. Large resistance causes the difference to be less marked.

R.M.S. Value of Non-Sinusoidal Waves

With a single harmonic, the equation is of the form

$$\text{Ordinate} = A \sin \omega t + B \sin n\omega t$$

A and B being maximum values, and n the number of the odd harmonic, then

Ordinate squared

$$= A^2 \sin^2 \omega t + 2AB \sin \omega t \sin n\omega t + B^2 \sin^2 n\omega t$$

Taking a large number of ordinates N, spaced equally over 1 cycle of the fundamental, and adding the N equations and dividing by N, the result is

Average of squared ordinates

$$= \text{av. of } A^2 \sin^2 \omega t + \text{av. of } B^2 \sin^2 n\omega t$$
$$+ \text{av. of } 2AB \sin \omega t \sin n\omega t$$
$$= \tfrac{1}{2}A^2 + \tfrac{1}{2}B^2 + 0$$

The average value of the last term, taken over a cycle for the fundamental, will be seen to be zero, if the curve is plotted.

Taking the square-root of both sides,

$$\text{R.M.S. value} = \sqrt{\frac{A^2}{2} + \frac{B^2}{2}}$$

With two harmonics,

Ordinate

$$= A \sin \omega t + B \sin n\omega t + C \sin m\omega t$$

Ordinate squared

$$= A^2 \sin^2 \omega t + B^2 \sin^2 n\omega t + C^2 \sin^2 m\omega t$$
$$+ 2AB \sin \omega t \sin n\omega t + 2BC \sin n\omega t \sin m\omega t$$
$$+ 2CA \sin \omega t \sin m\omega t$$

Proceeding as before,

$$\text{R.M.S. value} = \sqrt{\frac{A^2}{2} + \frac{B^2}{2} + \frac{C^2}{2}}$$

When the equation is of the form,

$$\text{Ordinate} = A \sin \omega t + B \sin (n\omega t + \theta)$$

$$\text{Ordinate squared} = A^2 \sin^2 \omega t + B^2 \sin^2 (n\omega t + \theta) + 2AB \sin \omega t \sin (n\omega t + \theta)$$

The average value of the last term is again zero when taken over a complete cycle of the fundamental, and the R.M.S. value is as found without the phase displacement of the harmonic.

Thus the reading on a voltmeter or ammeter is unaffected by the phase displacement of the harmonics; the wave-shape will be different, but the heating effect of the current will be the same.

Power with Non-Sinusoidal Wave-shapes

Let the volts be given by $(A \sin \omega t + B \sin n\omega t)$

and the current by $(C \sin (\omega t - \phi) + D \sin (n\omega t - \psi)$

Inst. watts

$$= \{A \sin \omega t + B \sin n\omega t\}\{C \sin (\omega t - \phi) + D \sin (n\omega t - \psi)\}$$

$$= AC \sin \omega t \sin (\omega t - \phi) + BD \sin n\omega t \sin (n\omega t - \psi)$$

$$+ AD \sin \omega t \sin (n\omega t - \psi) + BC \sin n\omega t \sin (\omega t - \phi)$$

The average value of the two last terms taken over a cycle of the fundamental is zero, so the average power depends on the sum of the average values of the two first terms. Inspection shows them to be the power of the fundamental current and volts, and the power of the harmonic current and its volts.

Hence, to find the power, sum the powers for each harmonic. Each harmonic has its own angle of lag, and there is now no such thing as a general angle of lag.

EXAMPLE. An alternator E.M.F. contains a third harmonic of amplitude 50 volts, the amplitude of the fundamental being 300 volts and its frequency 50. Find the reading on a voltmeter connected to its terminals.

$$\text{R.M.S. value} = \sqrt{\frac{A^2}{2} + \frac{B^2}{2}}$$

$$= \sqrt{\frac{300 \times 300}{2} + \frac{50 \times 50}{2}} = \sqrt{46250}$$

$$= 215 \text{ volts}$$

Without the harmonic the value would have been 212 volts.

EXAMPLE. Find the R.M.S. value of the current which the above alternator will send through a resistance of 5 ohms in series with a reactance of 5 ohms at 50 cycles.

The fundamental current will have a max. value

$$= \frac{300}{\sqrt{25 + 25}} = 42{\cdot}4 \text{ amp.}$$

The third harmonic current will have a max. value

$$= \frac{50}{\sqrt{25 + 225}} = 3{\cdot}16 \text{ amp.}$$

$$\text{The R.M.S. value} = \sqrt{\frac{(42{\cdot}4)^2}{2} + \frac{(3{\cdot}16)^2}{2}} = \sqrt{900 + 5}$$

$$= 30{\cdot}1 \text{ amp. (nearly)}$$

The power for the fundamental

$$= \tfrac{1}{2} \times 300 \times 42{\cdot}4 \times 0{\cdot}707 = 4500 \text{ watts}$$

and for the harmonic,

$$= \tfrac{1}{2} \times 50 \times 3{\cdot}16 \times 0{\cdot}31 = 24{\cdot}5 \text{ watts}$$

$\therefore$ the total power is 4524·5 watts

and the power factor, $\dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{4524{\cdot}5}{215 \times 30{\cdot}1} = 0{\cdot}7.$

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